1. INTERNATIONAL CONGRESS ON MATHEMATICS AND GEOMETRY

PROCEEDINGS BOOK

EDİTOR:
ASSOC. PROF. DR. SUAYIP YUZBASI

December 9, 2020
Ankara, Turkey

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1. INTERNATIONAL CONGRESS ON MATHEMATICS AND GEOMETRY

PROCEEDINGS BOOK

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Ankara, Turkey

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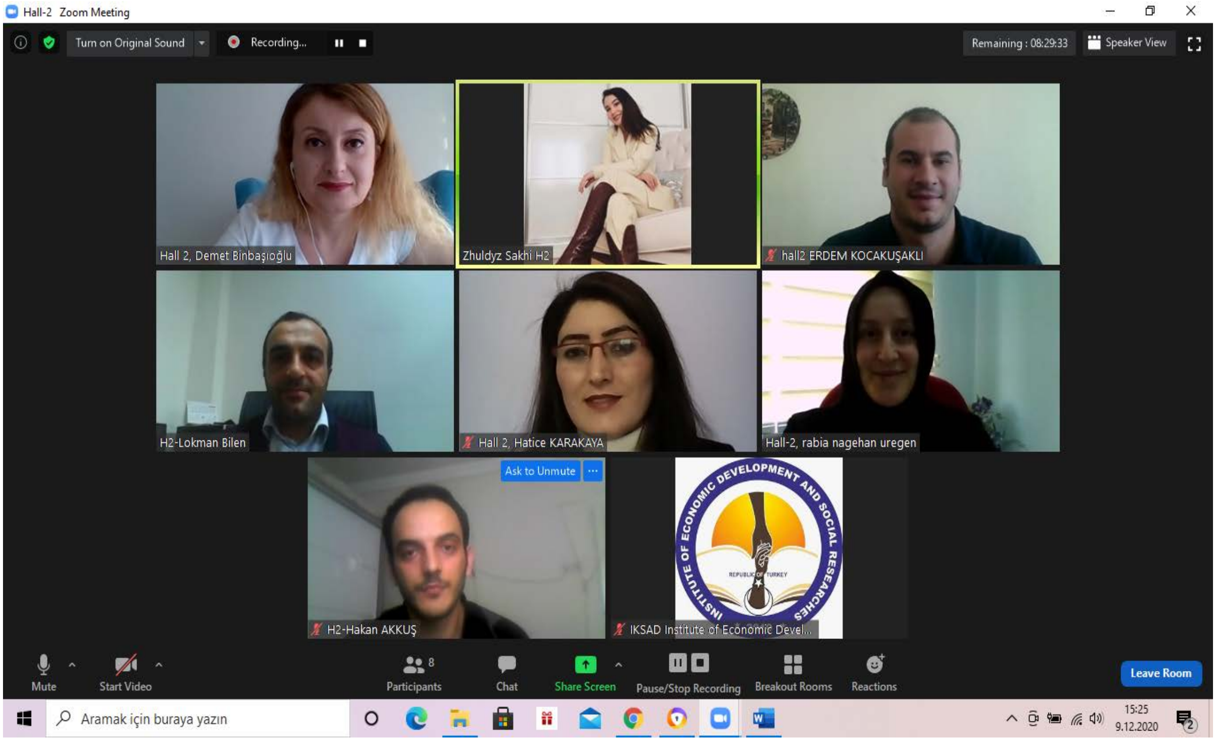

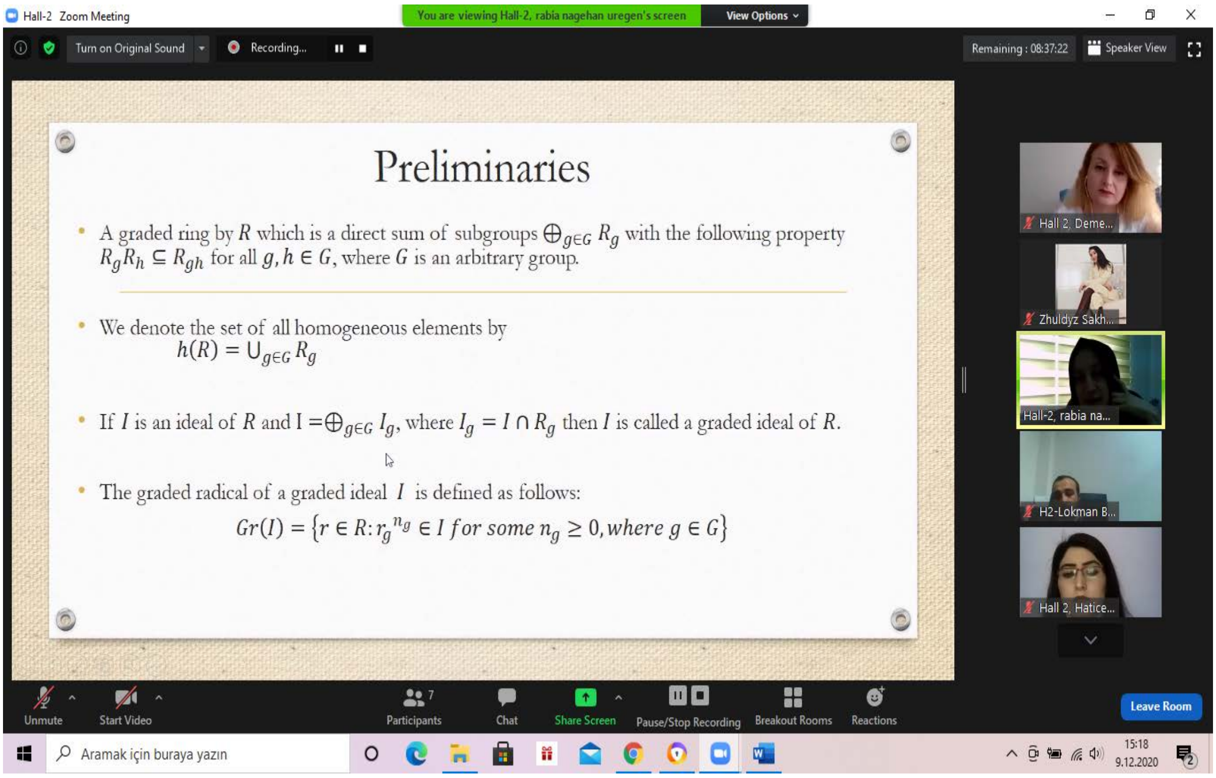

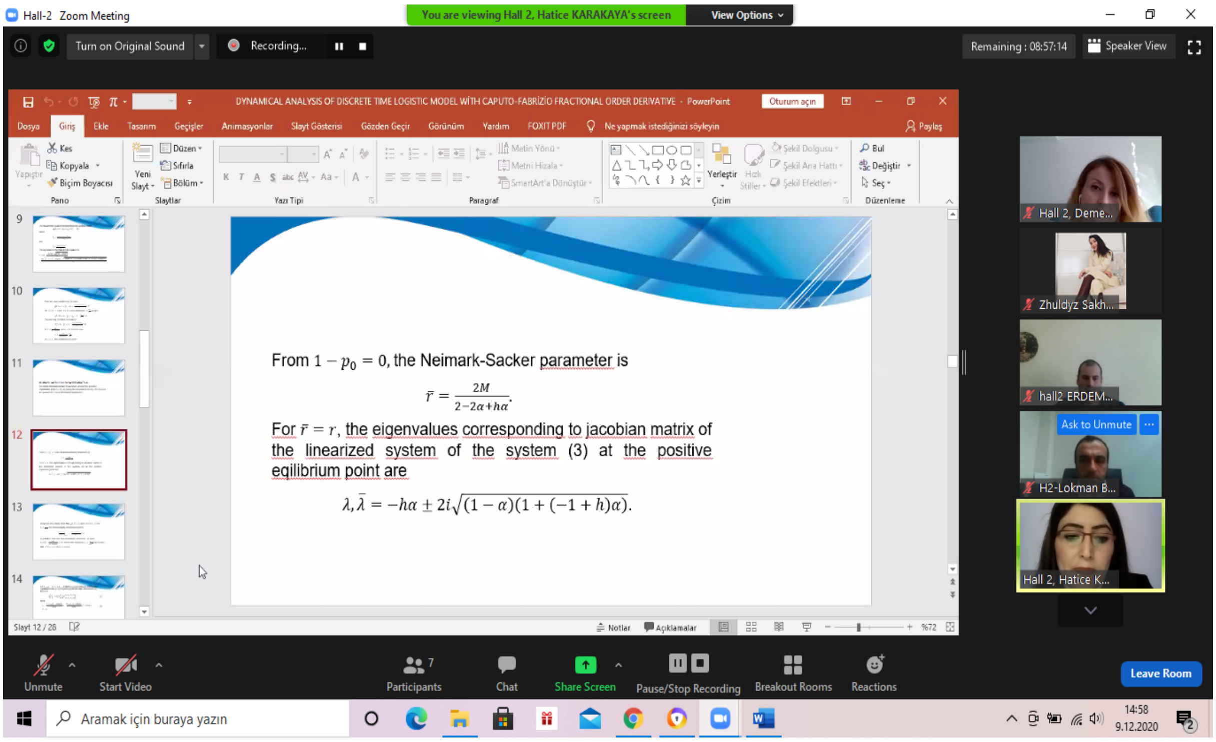
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DYNAMICAL ANALYSIS OF DISCRETE TIME LOGISTIC MODEL WITH CAPUTO-FABRIZIO FRACTIONAL ORDER DERIVATIVE - PowerPoint
From $1-p_0=0$, the Neimark-Sacker parameter is
$\bar{r}=\frac{2M}{2-2\alpha+h\alpha}$.
For $\bar{r}=r$, the eigenvalues corresponding to jacobian matrix of the linearized system of the system (3) at the positive eqilibrium point are
$\lambda,\bar{\lambda}=-h\alpha\pm 2i\sqrt{(1-\alpha)(1+(-1+h)\alpha)}$.
Slayt 12 / 28
Hall 2, Deme...
Zhuldyz Sakh...
hall2 ERDEM...
Ask to Unmute
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Hall 2, Hatice K...
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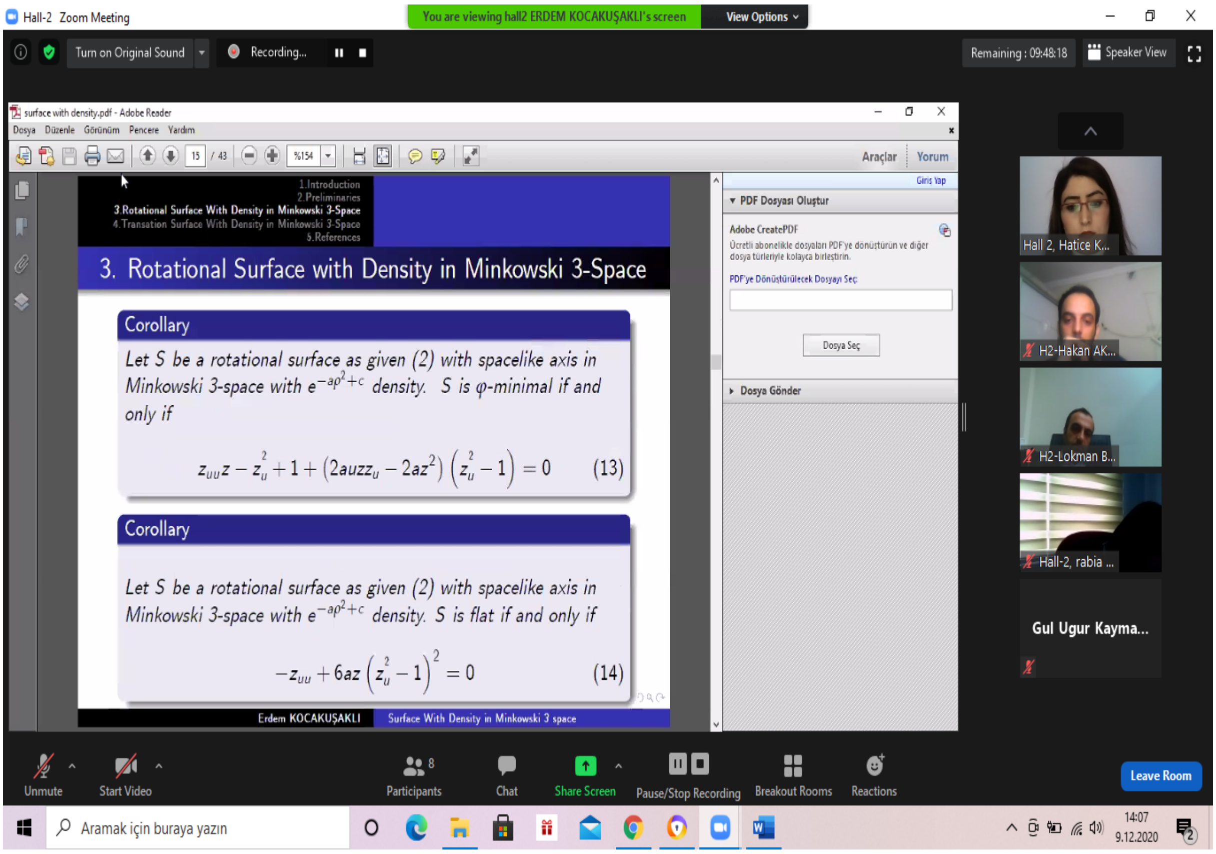
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surface with density.pdf - Adobe Reader
1.Introduction
2.Preliminaries
3.Rotational Surface With Density in Minkowski 3-Space
4.Translation Surface With Density in Minkowski 3-Space
5.References
3. Rotational Surface with Density in Minkowski 3-Space
Corollary
Let S be a rotational surface as given (2) with spacelike axis in Minkowski 3-space with $e^{-a\rho^2+c}$ density. S is φ-minimal if and only if
$z_{uu}z-z_u^2+1+\left(2auzz_u-2az^2\right)\left(z_u^2-1\right)=0$ (13)
Corollary
Let S be a rotational surface as given (2) with spacelike axis in Minkowski 3-space with $e^{-a\rho^2+c}$ density. S is flat if and only if
$-z_{uu}+6az\left(z_u^2-1\right)^2=0$ (14)
Erdem KOCAKUŞAKLI
Surface With Density in Minkowski 3 space
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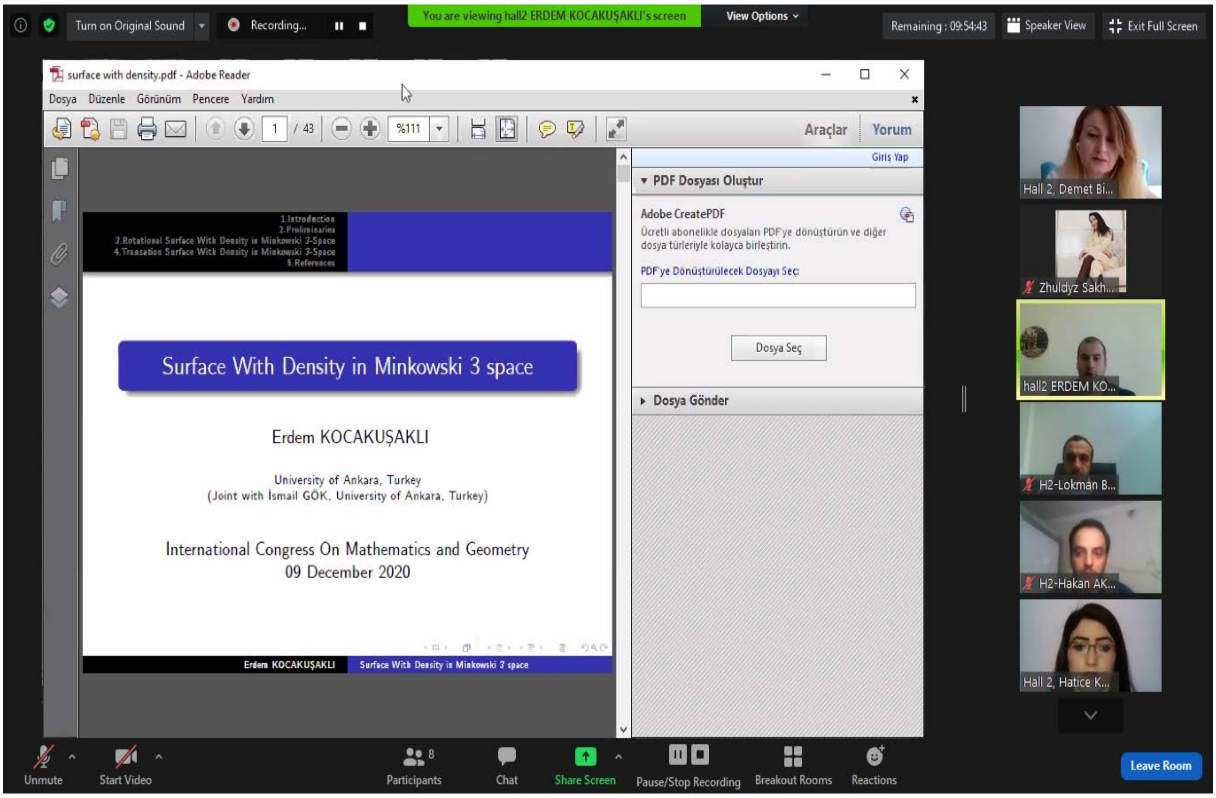
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2.Preliminaries
2.Rotational Surface With Density in Minkowski 3-Space
4.Translation Surface With Density in Minkowski 3-Space
5.References
Surface With Density in Minkowski 3 space
Erdem KOCAKUŞAKLI
University of Ankara, Turkey
(Joint with İsmail GÖK, University of Ankara, Turkey)
International Congress On Mathematics and Geometry
09 December 2020
Erdem KOCAKUŞAKLI
Surface With Density in Minkowski 3 space
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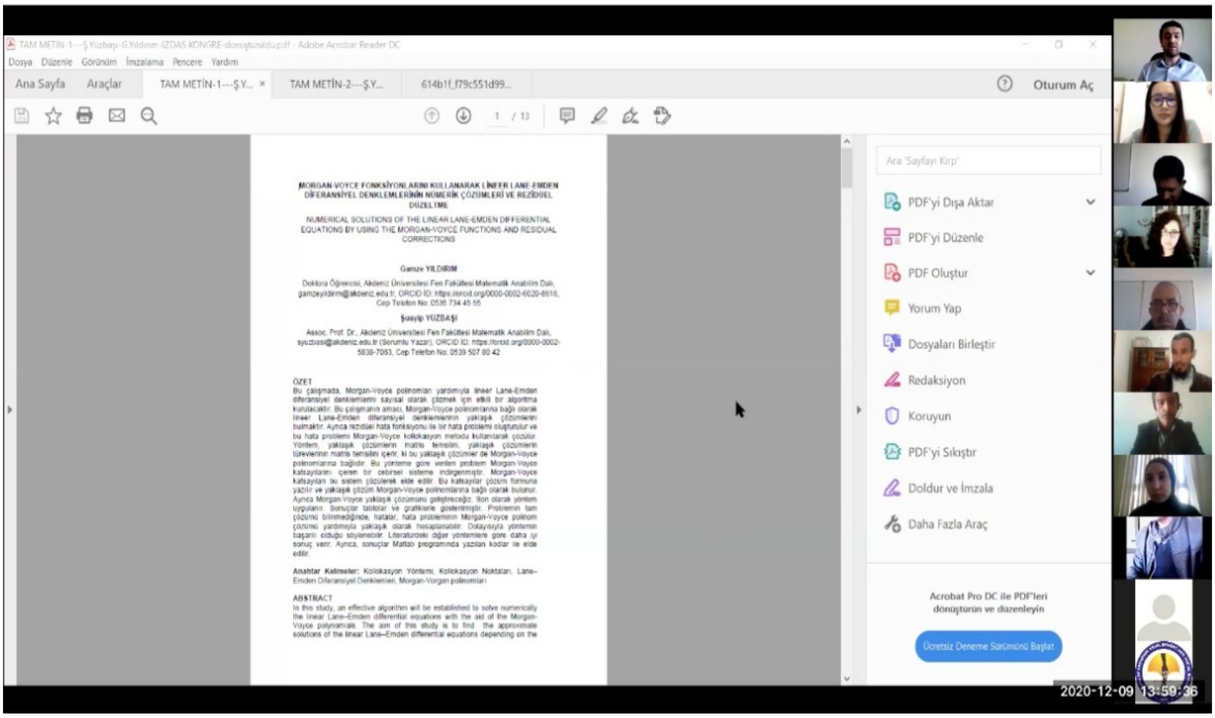
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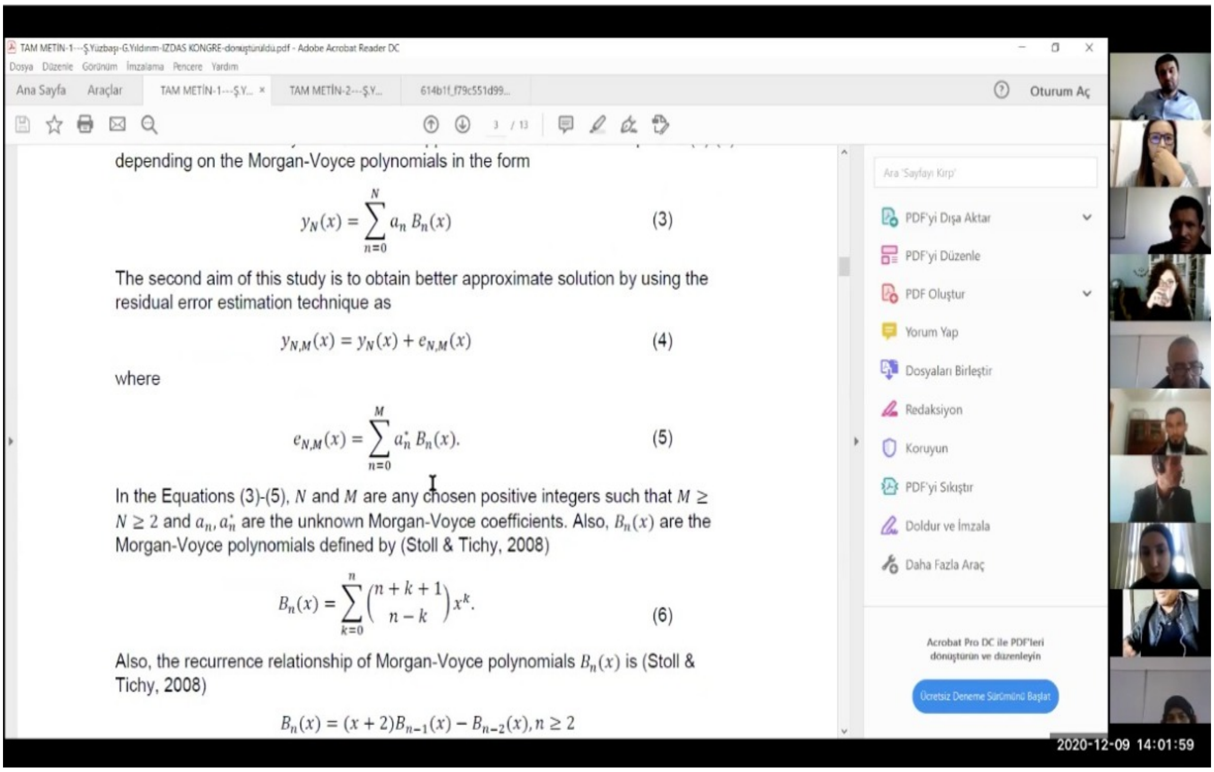
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Oturum Aç
depending on the Morgan-Voyce polynomials in the form
$$y_N(x) = \sum_{n=0}^{N} a_n B_n(x) \quad (3)$$
The second aim of this study is to obtain better approximate solution by using the residual error estimation technique as
$$y_{N,M}(x) = y_N(x) + e_{N,M}(x) \quad (4)$$
where
$$e_{N,M}(x) = \sum_{n=0}^{M} a_n^* B_n(x). \quad (5)$$
In the Equations (3)-(5), N and M are any chosen positive integers such that $M \geq N \geq 2$ and a_n, a_n^* are the unknown Morgan-Voyce coefficients. Also, $B_n(x)$ are the Morgan-Voyce polynomials defined by (Stoll & Tichy, 2008)
$$B_n(x) = \sum_{k=0}^{n} \binom{n+k+1}{n-k} x^k. \quad (6)$$
Also, the recurrence relationship of Morgan-Voyce polynomials $B_n(x)$ is (Stoll & Tichy, 2008)
$$B_n(x) = (x+2)B_{n-1}(x) - B_{n-2}(x), n \geq 2$$
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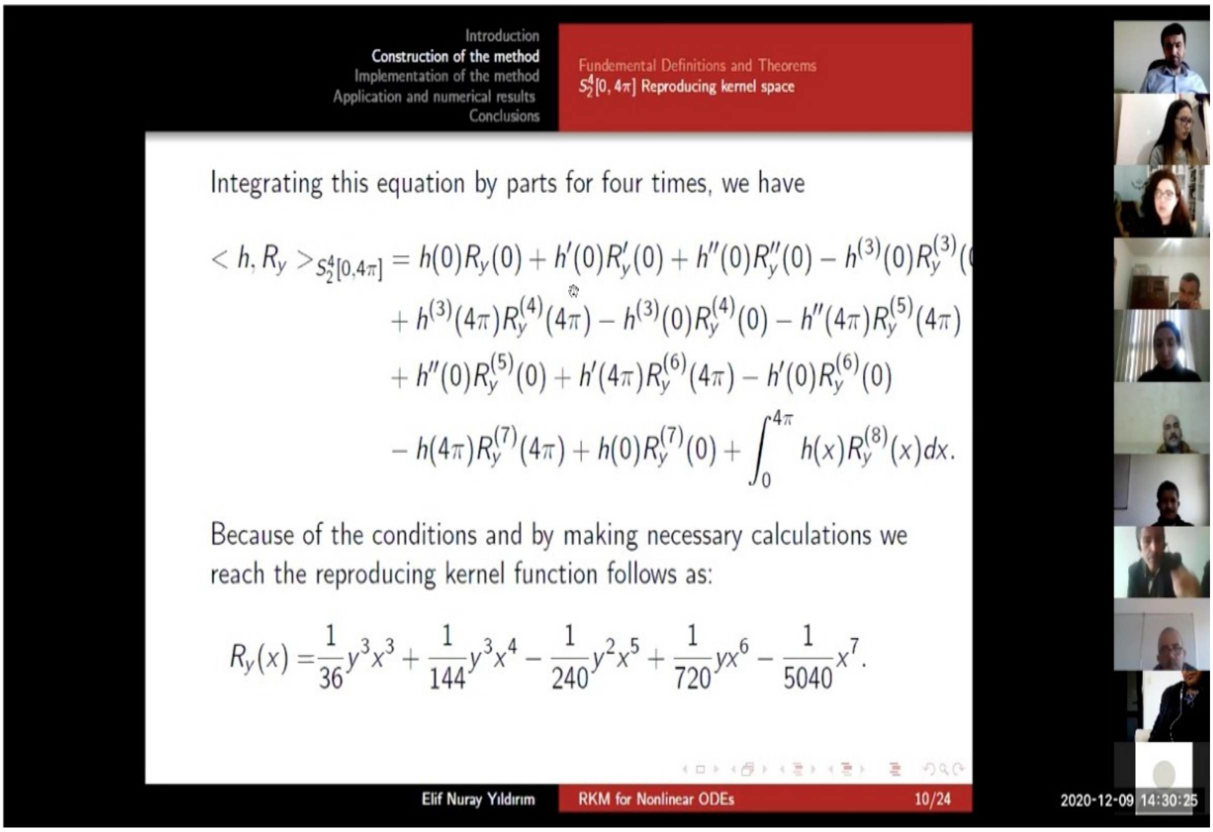
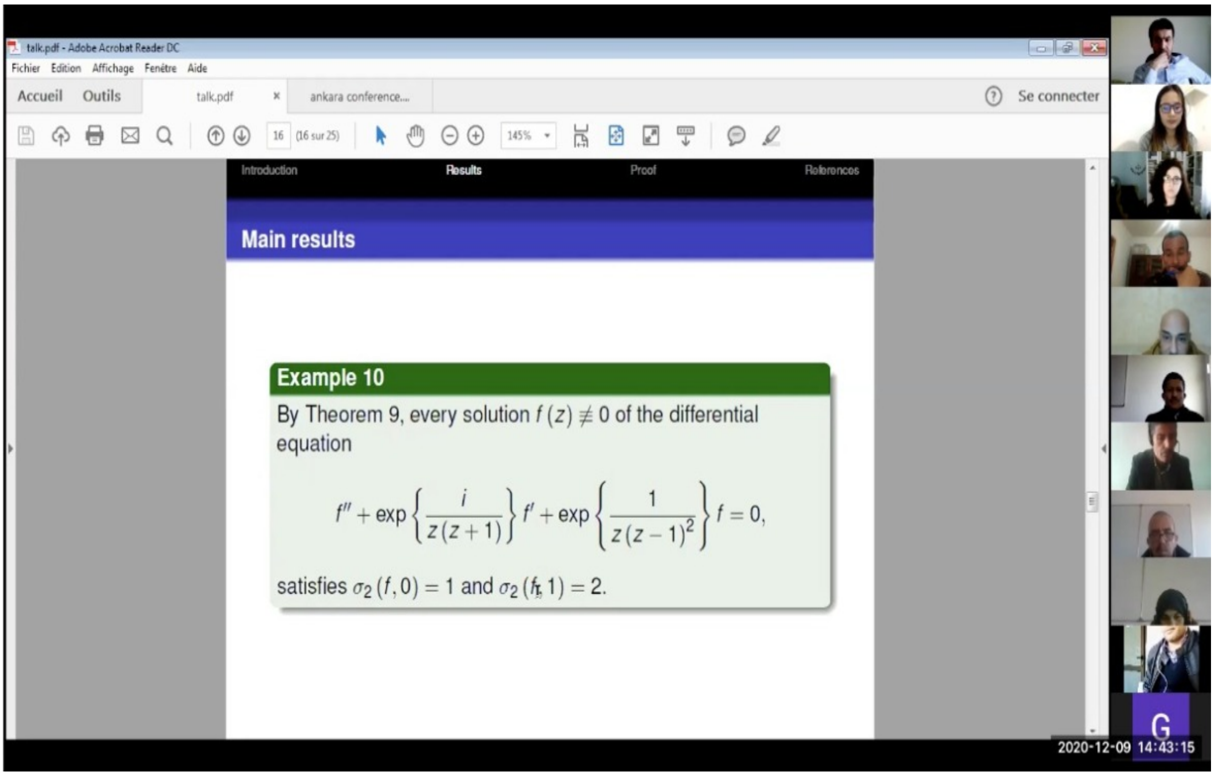

Introduction
Construction of the method
Implementation of the method
Application and numerical results
Conclusions
Fundemental Definitions and Theorems
$S_2^4[0, 4\pi]$ Reproducing kernel space
Integrating this equation by parts for four times, we have
$$< h, R_y >_{S_2^4[0,4\pi]} = h(0)R_y(0) + h'(0)R_y'(0) + h''(0)R_y''(0) - h^{(3)}(0)R_y^{(3)}(+ h^{(3)}(4\pi)R_y^{(4)}(4\pi) - h^{(3)}(0)R_y^{(4)}(0) - h''(4\pi)R_y^{(5)}(4\pi) + h''(0)R_y^{(5)}(0) + h'(4\pi)R_y^{(6)}(4\pi) - h'(0)R_y^{(6)}(0) - h(4\pi)R_y^{(7)}(4\pi) + h(0)R_y^{(7)}(0) + \int_0^{4\pi} h(x)R_y^{(8)}(x)dx.$$
Because of the conditions and by making necessary calculations we reach the reproducing kernel function follows as:
$$R_y(x) = \frac{1}{36}y^3x^3 + \frac{1}{144}y^3x^4 - \frac{1}{240}y^2x^5 + \frac{1}{720}yx^6 - \frac{1}{5040}x^7.$$
Elif Nuray Yıldırım
RKM for Nonlinear ODEs
10/24
2020-12-09 14:30:25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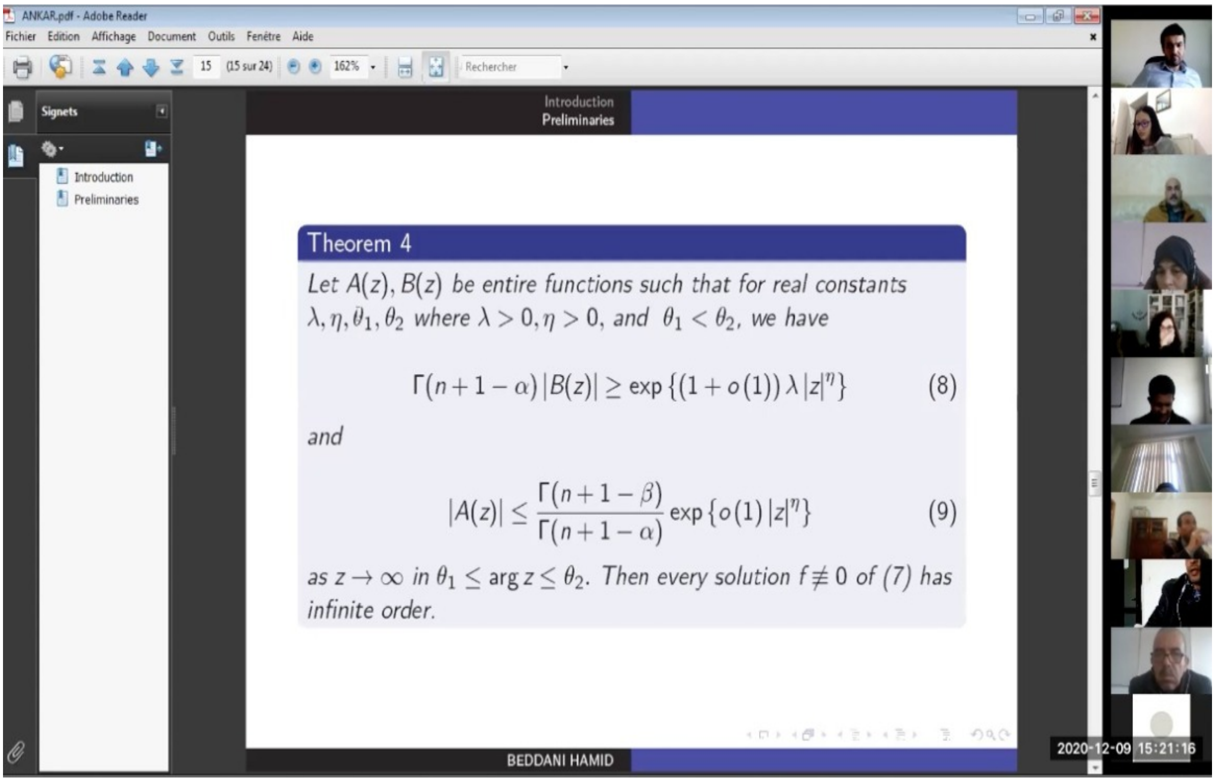
Introduction Results Proof References
Main results
Example 10
By Theorem 9, every solution $f(z) \not\equiv 0$ of the differential equation
$$f'' + \exp\left\{\frac{i}{z(z+1)}\right\} f' + \exp\left\{\frac{1}{z(z-1)^2}\right\} f = 0,$$
satisfies $\sigma_2(f,0) = 1$ and $\sigma_2(f,1) = 2$.
2020-12-09 14:43:15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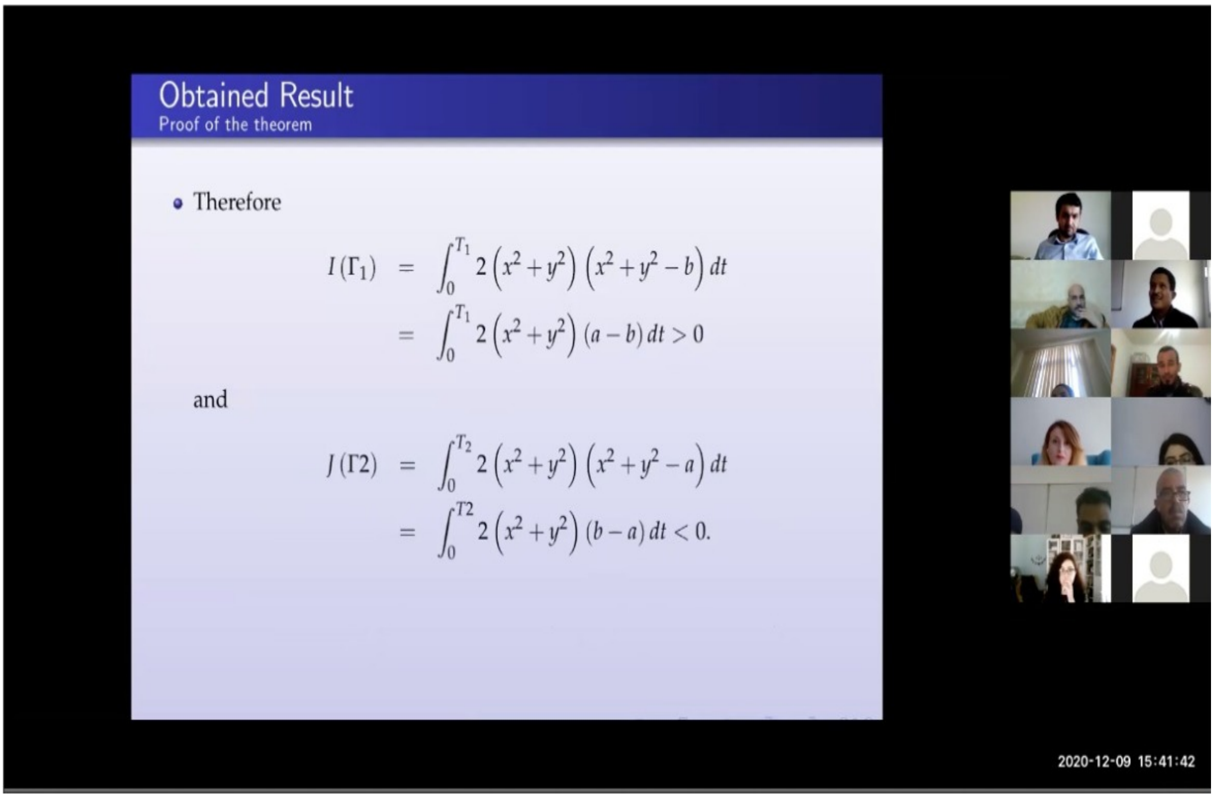
Introduction
Preliminaries
Introduction
Preliminaries
Theorem 4
Let $A(z), B(z)$ be entire functions such that for real constants $\lambda, \eta, \theta_1, \theta_2$ where $\lambda > 0, \eta > 0$, and $\theta_1 < \theta_2$, we have
$$\Gamma(n+1-\alpha)\,|B(z)| \geq \exp\{(1+o(1))\,\lambda\,|z|^{\eta}\} \quad (8)$$
and
$$|A(z)| \leq \frac{\Gamma(n+1-\beta)}{\Gamma(n+1-\alpha)} \exp\{o(1)\,|z|^{\eta}\} \quad (9)$$
as $z \to \infty$ in $\theta_1 \leq \arg z \leq \theta_2$. Then every solution $f \not\equiv 0$ of (7) has infinite order.
BEDDANI HAMID
2020-12-09 15:21:16


Obtained Result
Proof of the theorem
Therefore
$I(\Gamma_1) = \int_0^{T_1} 2\left(x^2+y^2\right)\left(x^2+y^2-b\right)dt = \int_0^{T_1} 2\left(x^2+y^2\right)(a-b)\,dt > 0$
and
$J(\Gamma 2) = \int_0^{T_2} 2\left(x^2+y^2\right)\left(x^2+y^2-a\right)dt = \int_0^{T2} 2\left(x^2+y^2\right)(b-a)\,dt < 0.$
2020-12-09 15:41:42


2020-12-09 16:00:02

1. INTERNATIONAL CONGRESS ON MATHEMATICS AND GEOMETRY

9 DECEMBER 2020

ANKARA

CONGRESS PROGRAM

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SESSION-1, HALL-1/OTURUM-1, SALON-1

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AUTHOR	AFFILITION	Title
Gamze YILDIRIM **Assoc. Prof. Dr. Şuayip YÜZBAŞI**	*Akdeniz University*	NUMERICAL SOLUTIONS OF THE LINEAR LANE-EMDEN DIFFERENTIAL EQUATIONS BY USING THE MORGAN-VOYCE FUNCTIONS AND RESIDUAL CORRECTIONS
Gamze YILDIRIM **Assoc. Prof. Dr. Şuayip YÜZBAŞI**	*Akdeniz University*	AN OPERATIONAL MATRIX METHOD FOR THE SYSTEMS OF HIGHER-ORDER LINEAR FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS BY USING THE PELL-LUCAS FUNCTIONS
Arş. Gör. Elif NURAY YILDIRIM	*Istanbul Commerce University*	ON SOLUTIONS OF A HIGHER ORDER NONHOMOGENEOUS ORDINARY DIFFERENTIAL EQUATION WITH a NUMERICAL METHOD
Saada HAMOUDA	*University of Mostaganem, Algeria*	LOGARITHMIC DERIVATIVE NEAR A SINGULAR POINT AND APPLICATIONS IN LINEAR DIFFERENTIAL EQUATIONS
Dr. FETTOUCH Houari	*University of Mostaganem, Algeria*	GROWTH OF LOCAL SOLUTIONS TO LINEAR DIFFERENTIAL EQUATIONS AROUND AN ISOLATED ESSENTIAL SINGULARITY
Dr. Habib DJOURDEM	*University of Oran1, Ahmed Benbella. Algeria*	EXISTENCE RESULT OF A FRACTIONAL DIFFERENTIAL EQUATION OF HADAMARD TYPE WITH INTEGRAL BOUNDARY VALUE CONDITIONS
Dr. BEDDANI HAMID	*University of Mostaganem, Algeria*	ORDERS OF SOLUTIONS OF FRACTIONAL DIFFERENTIAL EQUATION
Samira HAMANI **Amouria HAMMOU** **Johnny HENDERSON**	*Universit_e de Mostaganem* *Baylor University*	IMPULSIVE FRACTIONAL DIFFERENTIAL EQUATIONS INVOLVING THE HADAMARD FRACTIONAL DERIVATIVE
Mohamed GRAZEM	*University of Boumerdes, Algeria*	COEXISTENCE OF TWO LIMIT CYCLES FOR A CLASS OF PLANAR DIFFERENTIAL SYSTEMS
Dr. Naveen Gupta	*Lovely Professional University Phagwara, India*	OPTICAL PHASE CONJUGATION AND ITS APPLICATIONS
Dr.Noureddine BOUTERAA	*University of Oran1, Ahmed Benbella. Algeria*	STABILITY OF SOLUTIONS OF INITIAL VALUE PROBLEM FOR A CLASS OF STOCHASTIC FRACTIONAL DIFFERENTIAL EQUATION WITH NOISE

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Dr. Erdem KOCAKUŞAKLI	*Ankara University*	TUBULAR SURFACES CONSTRUCTED BY SPHERICAL INDICATRICES WITH DENSITY
Dr. Öğr. Üyesi Demet BİNBAŞIOĞLU	*Gaziosmanpasa University*	SOME COMMON FIXED POINT THEOREMS IN Ƒ-METRIC SPACES
Dr. Öğr. Üyesi Lokman BİLEN	*Iğdır University*	CONFORMAL AND HOLOMORPHICALLY PROJECTIVE VECTOR FIELDS ON COTANGENT BUNDLES
Hatice KARAKAYA **Doç. Dr. Şenol KARTAL**	*Erciyes University* *Nevsehir Hacı Bektas Veli University*	AYRIK ZAMANLI CAPUTO-FABRİZİO KESİRSEL MERTEBEDEN LOJİSTİK MODELİN DİNAMİK ANALİZİ
Hakan AKKUŞ **Dr. Öğr. Üyesi Rabia Nagehan ÜREGEN** **Prof. Dr. Engin ÖZKAN**	*Erzincan Binali Yıldırım University*	CATALAN TRANSFORM OF THE K-JACOBSTHAL-LUCAS SEQUENCE
Dr. Öğr. Üyesi Rabia Nagehan ÜREGEN	*Erzincan Binali Yıldırım Üniversitesi*	A NOTE ON A GENERALIZATION OF GRADED PRIME IDEALS
Assisit. Prof. Dr. Valdete Loku **Prof. Dr. Naim L. Braha**	*University of Applied Sciences Ferizaj, Kosova* *University of Prishtina, Kosova*	STATISTICAL KOROVKIN AND VORONOVSKAYA TYPE THEOREM FOR THE CESARO SECOND-ORDER OPERATOR OF FUZZY NUMBERS
Assoc. Prof. Dr. ANNA NEENA GEORGE	*GVM's Dr. Dada Vaidya College of Education, India*	MATHEMATICS EDUCATION CREATING FEAR AND MISCONCEPTION
Ryan Jay Bernales Gumban **Denis Abao Tan**	*Malinao High School Extension- Gastav Campus, Philippines* *Central Mindanao University, Philippines*	STUDENTS' MATHEMATICS PERFORMANCE, ENGAGEMENT AND INFORMATION AND COMMUNICATION TECHNOLOGY COMPETENCIES IN AFLIPPED CLASSROOM ENVIRONMENT

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PROCEEDINGS BOOK

AYRIK ZAMANLI CAPUTO-FABRİZİO KESİRSEL MERTEBEDEN LOJİSTİK MODELİN DİNAMİK ANALİZİ

DYNAMICAL ANALYSIS OF DISCRETE TIME LOGISTIC MODEL WITH CAPUTO-FABRİZİO FRACTIONAL ORDER DERIVATIVE

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ÖZET

Bu çalışmada, helianthus bitkisinin yıllık büyüme oranını tanımlayan kesirsel mertebeden Caputo-Fabrizio lojistik modeli ele alınmıştır. Modele ilk olarak İki Adımlı Adams-Bashforth Metoduna dayalı bir ayrıklaştırma işlemi uygulanmış ve ardından bir fark denklem sistemi elde edilmiştir. Ayrık sistemin pozitif denge noktasının kararlılık koşulları, Schur-Cohn kriteri kullanılarak belirlenmiştir. Dahası, çatallanma analizi ile ayrık sistemin pozitif denge noktası civarında Neimark-Sacker çatallanması olduğu gösterilmiştir. Neimark-Sacker çatallanmasının yönü ve kararlılığı, normal form ve merkez manifold teorisi kullanılarak belirlenmiştir. Ayrıca maksimum Lyapunov üstellerinin hesaplanmasıyla sistemde kaotik davranışların varlığı araştırılmıştır. Parametre değerleri, biyolojik gerçeklerle uyumlu olması için literatürde verilen deneysel verilerden seçilmiştir. Son olarak, teorik sonuçların doğruluğunu göstermek için nümerik simülasyonlar kullanılmıştır.

Anahtar Kelimeler: Caputo-Fabrizio Kesirsel Merebeden Türev, Lojistik Diferansiyel Denklem, İki Adımlı Adams-Bashfort Metodu, Neimark-Sacker Çatallanması

ABSTRACT

In this paper, the Caputo-Fabrizio fractional order logistic model which describes annual growth rate of the helianthus plant is considered. Firstly, a discretization process based on Two Step Adams-Bashforth Method is applied to the model and then we obtain a system of difference equations. Stability conditions of positive equilibrium point of the discrete system are determined by using Schur-Cohn criterion. Morever, we also deal with the bifurcation analysis and show that the discrete system undergoes Neimark-Sacker bifurcation around the positive equilibrium point. The direction and stability of the Neimark-Sacker bifurcation are determined by using the normal form and center manifold theory. We also investigate the chaotic behavior of the system by calculating the maximum Lyapunov exponents. Parameter values are selected from experimental data that is given in the literature in order to be compatible with biological fact. Finally, numerical simulations are used to demonstate the accuracy of theoretical result.

Keywords: Caputo-Fabrizio Fractional Order Derivative, Logistic Differential Equation, Two Step Adams-Bashforth Method, Neimark-Sacker Bifurcation

K-JACOBSTHAL-LUCAS DİZİSİNİN KATALAN DÖNÜŞÜMÜ

CATALAN TRANSFORM OF THE K-JACOBSTHAL-LUCAS SEQUENCE

Hakan AKKUŞ

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ÖZET

Bu çalışmada k-Jacobsthal-Lucas dizisinin $S_{k,n}$ catalan dönüşümünün C$S_{k,n}$ tanımlanmıştır.k-Jacobsthal-Lucas dizisinin $S_{k,n}$ catalan dönüşümünü elde edilmiştir. Ayrıca C$S_{k,n}$dönüşümü alt üçgen matris olan Catalan matrisi C ile n x 1 tipindeki S_k matrisinin çarpımı olarak yazılmıştır ve bazı k-Jacobsthal-Lucas sayılarının Hankel dönüşümü bulunmuştur.

Anahtar Kelimeler: k-Pell dizisi, k- Lucas dizisi, k-Fibonacci dizisi, Catalan Dönüşümü, Hankel Dönüşümü

ABSTRACT

In this study, $CS_{k,n}$ of $S_{k,n}$ Catalan transformation of $k-$Jacobsthal-Lucas sequence is defined. $S_{k,n}$ Catalan transformation of $k-$Jacobsthal-Lucas $S_{k,n}$ sequences is obtained.In addition the transformation of C$S_{k,n}$ is written as the product of the Catalan matrix C, which is the lower triangular matrix, and the S_k matrix of type $n\ x$ 1, and the Hankel transformations of some k-Jacobsthal-Lucas numbers has been found.

Keywords: k-Pell sequences, k-Lucas sequences, k-Fibonacci sequences, Catalan Transform, Hankel Transform

KAYNAKLAR

[1] Barry, P. 2005. ‘’A Catalan transform and related transformations on integer sequences’’. J.Integer Seq., 8(4), 1-24.

[2] Falcon, S. (2013). ‘’Catalan Transform of the *k*-Fibonacci sequence’’. Commun. Korean Math. Soc. 28(4), 827–832.

[**3**] Falc´on, S., and Plaza, A. 2007. ‘’On the Fibonacci k-numbers’’, Chaos Solitons Fractals 32(5), 1615–1624.

[**4**] Koshy, T. 2001. ‘’Fibonacci and Lucas Numbers with Applications’’. New York, NY: Wiley-Interscience Publication.

SOME COMMON FIXED POINT THEOREMS IN Ƒ-METRIC SPACES

Ƒ-METRİK UZAYLARDA BAZI ORTAK SABİT NOKTA TEOREMLERİ

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ÖZET

Son zamanlarda, Jleli ve Samet tarafından Ƒ metrik uzay kavramı tanıtılmış ve bu uzayların doğal topolojisi tanımlanmıştır. Ayrıca bu uzaylarda Banach sabit nokta prensibinin yeni bir versiyonu verilmiştir. Bu çalışmamızda Ƒ metrik uzaylarda bazı ortak sabit nokta teoremleri kanıtlanmıştır.

Anahtar Kelimeler : Ƒ Metrik Uzay, Büzülme Dönüşümü, Ortak Sabit Nokta Teoremi

ABSTRACT

Lately, the concept of Ƒ metric space has been introduced and have been defined a natural topology in this spaces by Jleli and Samet. Also a new version of Banach contraction principle has been given in the Ƒ metric spaces. In this paper, we prove some common fixed point theorems in the spaces.

Keywords: Ƒ Metric Space, Contraction Mapping, Common Fixed Point Theorem

STUDENTS' MATHEMATICS PERFORMANCE, ENGAGEMENT AND INFORMATION AND COMMUNICATION TECHNOLOGY COMPETENCIES IN A FLIPPED CLASSROOM ENVIRONMENT

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ABSTRACT

An investigation was conducted to ascertain the effectiveness of the Flipped Classroom in the mathematics performance, engagement, and ICT competencies of the Grade 9 students of Malinao High School Extension- Gastav Campus. Specifically, the study sought to: (1) identify the level of mathematics performance of the students when exposed to flipped classroom; (2) determine the level of students' engagement in Mathematics with the use of flipped classroom; (3) ascertain the level of students' ICT competencies with the use of flipped classroom; (4) differentiate the level of Mathematics performance of the students with the integration of flipped classroom; (5) find out if there is a significant difference in students' level of engagement in Mathematics with the integration of flipped classroom; (6) distinguish if there a significant difference in students' ICT Competencies with the integration of flipped classroom.

A one shot pretest-posttest was conducted to assess the effectiveness of the flipped Classroom. Results showed that students exposed to Flipped classroom have significantly higher performance in terms of posttest and retention test scores. Also, a significant difference in the students' mathematics engagement and ICT competencies before and after the intervention was found. Students gained basic knowledge of ICT competencies based on the increase in the over-all mean scores from the pre-test to post test. They acquired basic skills after exposure in a Flipped classroom. Moreover, there was a significant difference in the affective and cognitive engagement of students in Mathematics when exposed to the flipped classroom. Students were significantly engaged in both cognitive and affective aspects while learning Mathematics.

OPTICAL PHASE CONJUGATION AND ITS APPLICATIONS

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ABSTRACT

This paper presents a review on a novel nonlinear effect known as optical phase conjugation. Emphasis is put on providing fundamental aspects of this phenomenon by avoiding complicated mathematics. Various methods like four wave mixing and stimulated Brillouin scattering to produce optical phase conjugation have been discussed in detail. Various applications of this phenomenon also have been discussed.

Keywords: Phase conjugation, Stimulated Brillouin scattering, Four wave mixing.

KOTANJANT DEMETTE KONFORMAL VE HOLOMORFİK PROJEKTİF VEKTÖR ALANLARI

CONFORMAL AND HOLOMORPHICALLY PROJECTIVE VECTOR FIELDS ON COTANGENT BUNDLES

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ÖZET

(M,∇) diferensiyellenebilir bir manifold, ∇, M üzerinde bir lineer konneksiyon ve T^*M de (M,∇) nın ${}^R\nabla$ Riemann genişlemesi metriğine sahip kotanjant demeti olsun. Sunulan bu çalışmada kotanjant demette Riemann genişlemesi metriğine göre konformal ve holomorfik projektif vektör alanlarının detaylı sınıflandırması ve bu vektör alanları ile ilgili bazı geometrik sonuçlar verilmiştir.

Anahtar Kelimeler: Konformal vektör alanı, holomorfik projektif vektör alanı, Riemann genişlemesi.

ABSTRACT

Let (M,∇) be a differentiable manifold with lineer connection ∇ and T^*M it's cotangent bundle with Riemannian extension metric ${}^R\nabla$. In the present paper we presents detailed classification of conformal and holomorphically projective vector fields on cotangent bundle with respect to the Riemannian extension and some geometric results related to them.

Keywords: Conformal vector field, holomorphically projective vector field, Riemannian extension.

GROWTH OF SOLUTIONS OF A CLASS OF LINEAR DIFFERENTIAL EQUATIONS AROUND AN ISOLATED ESSENTIAL SINGULARITY

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ABSTRACT

In this paper we study the growth of solutions of certain class of linear differential equations around an isolated essential singularity point. For that, we transform by making use a conformal mapping certain results from the complex plane to a neighborhood of a singular point. We will see that there are a large similarities between the complex plane results and this investigation.

INTRODUCTION AND STATEMENT OF RESULTS

Throughout this paper, we assume that the reader is familiar with the fundamental results and the standard notations of the Nevanlinna value distribution theory of meromorphic function on the complex plane $\mathbb{C}$ and in the unit disc $D=\{z\in\mathbb{C}:\ |z|<1\}$ (see [haym, yang]). The importance of this theory has inspired many authors to find modifications and generalizations to different domains. Extensions of Nevanlinna Theory to annuli have been made by [bieb, khri, kond, korh, mark]. In this paper, we concentrate our investigation near an isolated essential singular point. We start to give the appropriate definitions. Set $\overline{\mathbb{C}}=\mathbb{C}\cup\{\infty\}$ and suppose that $f(z)$ is meromorphic in , where . Define the counting function of by

$$N_{z_0}(r,f)=-\int_{\infty}^{r}\frac{n(t,f)-n(\infty,f)}{t}dt-n(\infty,f)\log r,$$

where $n(t,f)$ counts the number of poles of $f(z)$ in the region $\{z\in\mathbb{C}:\ t\le|z-z_0|\}\cup\{\infty\}$ each pole according to its multiplicity; and the proximity function by

$$m_{z_0}(r,f)=\frac{1}{2\pi}\int_{0}^{2\pi}\ln^{+}\left|f\left(z_0-re^{i\varphi}\right)\right|d\varphi.$$

The characteristic function of is defined in the usual manner by

$$T_{z_0}(r,f)=m_{z_0}(r,f)+N_{z_0}(r,f).$$

In addition, the order of meromorphic function $f(z)$ near z_0 is defined by

$$\sigma_T(f,z_0)=\limsup_{r\to 0}\frac{\log^+ T_{z_0}(r,f)}{-\log r}.$$

For an analytic function $f(z)$ in $\overline{\mathbb{C}}-\{z_0\}$, we have also the definition

$$\sigma_M(f,z_0)=\limsup_{r\to 0}\frac{\log^+\log^+ M_{z_0}(r,f)}{-\log r},$$

where $M_{z_0}(r,f)=\max\{|f(z)| \; : \; |z-z_0|=r\}$.

For example, the function $f(z)=\exp\left\{\frac{1}{(z_0-z)^n}\right\}$, where $n\in\mathbb{N}\setminus\{0\}$, we have

$M_{z_0}(r,f)=\exp\left\{\frac{1}{r^n}\right\}$, and then $\sigma_M(f,z_0)=n$. We have also

$T_{z_0}(r,f)=m_{z_0}(r,f)=\frac{1}{2\pi}\int_0^{2\pi}\ln^+\left|f\left(z_0-re^{i\varphi}\right)\right|d\varphi=\frac{1}{r^n}$, and so $\sigma_T(f,z_0)=n$.

For the function $f(z)=\exp\left\{\frac{-1}{(1-z)}\right\}$, we have $\sigma(f,1)=1$ while in the unit disc we have $\sigma_T(f)=\sigma_M(f)=0$.

We see that in the unit disc we have $\sigma_T(f)\le\sigma_M(f)\le\sigma_T(f)+1$ and in the complex plane we have $\sigma_T(f)=\sigma_M(f)$. Now, how about the relation between $\sigma_T(f,z_0)$ and $\sigma_M(f,z_0)$? Below, in Lemma lem2, we will prove that if $f(z)$ is meromorphic function in $\overline{\mathbb{C}}-\{z_0\}$ and $g(w)=f\left(z_0-\frac{1}{w}\right)$, then $g(w)$ is a meromorphic function in $\mathbb{C}$ and we have $T(R,g)=T_{z_0}(r,f)$; where $R=\frac{1}{r}$; which implies that $\sigma_T(f,z_0)=\sigma_M(f,z_0)$. So, we can use the notation $\sigma(f,z_0)$ without any ambiguity.

By the usual manner, we define the hyper order near as follows:

$$\sigma_{2,T}(f,z_0)=\limsup_{r\to 0}\frac{\log^+\log^+ T_{z_0}(r,f)}{-\log r},$$

$$\sigma_{2,M}(f,z_0)=\limsup_{r\to 0}\frac{\log^+\log^+\log^+ M_{z_0}(r,f)}{-\log r}.$$

The linear differential equation

$$f''+A(z)e^{az}f'+B(z)e^{bz}f=0,$$

where $A(z)$ and $B(z)$ are entire functions, is investigated by many authors; see for example [ozaw,chen1,chen2,gund2]. In [chen1], Chen proved that if $ab \neq 0$ and $\arg a \neq \arg b$ or $a = cb$ $(0 < c < 1 \text{ or } c > 1)$, then every solution $f(z) \not\equiv 0$ of (1) is of infinite order. Recently, the second author proved results similar to (1) in the unit disc concerning the differential equation

$$f'' + A(z)e^{\frac{a}{(z_0-z)^{\mu}}} f' + B(z)e^{\frac{b}{(z_0-z)^{\mu}}} f = 0,$$

where $A(z)$ and $B(z)$ are analytic in the unit disc, $\mu > 0$ and $\arg a \neq \arg b$ or $a = cb$ $(0 < c < 1)$, see [ham12]. However, the method of [ham12] does not work in general for the case $0 < \mu \leq 1$: see the discussion in [ham12]. The case $\mu = 1$ will be investigated in the following theorem with certain modifications on $A(z)$ and $B(z)$.

Theorem *Let z_0, a, b be complex constants such that $\arg a \neq \arg b$ or $a = cb$ and be a positif integer. Let $A(z), B(z) \not\equiv 0$ be analytic functions in $\overline{\mathbb{C}} - \{z_0\}$ with $\max\{\sigma(A, z_0), \sigma(B, z_0)\} < n$. Then, every solution $f(z) \not\equiv 0$ of the differential equation*

$$f'' + A(z)e^{\frac{a}{(z_0-z)^{n}}} f' + B(z)e^{\frac{b}{(z_0-z)^{n}}} f = 0.$$

satisfies $\sigma(f, z_0) = \infty$ with $\sigma_2(f, z_0) = n$.

In [frei], Frei proved the following result in the complex plane.

Theorem *[frei] If the differential equation*

$$g'' + e^{-w} g' + cg = 0$$

where $c \neq 0$ is a complex constant, possesses a solution $g \not\equiv 0$ of finite order, then $c = -k^2$ where is a positive integer. Conversely, for each positive integer k, the equation (eq1) with $c = -k^2$, possesses a solution g which is a polynomial in e^w of degree k.

The analogous of this results, near a singular point , is as the following.

Theorem *Let $c \neq 0,\ z_0$ be complex numbers. If the differential equation*

$$f'' + \left(\frac{1}{(z_0 - z)^2} e^{\frac{-1}{(z_0-z)}} - \frac{2}{(z_0 - z)} \right) f' + \frac{c}{(z_0 - z)^4} f = 0$$

possesses a solution $f(z) \not\equiv 0$ of finite order $\sigma(f, z_0) < \infty$ then $c = -k^2$, where is an *integer. Conversely, for each positive integer , the equation (eq2) with ,*

possesses a solution which is a polynomial in of degree .

Example $f_1(z)=1+e^{\frac{1}{(z_0-z)}}$ *is a solution of the differential equation*

$$f''+\left(\frac{1}{(z_0-z)^2}e^{\frac{-1}{(z_0-z)}}-\frac{2}{(z_0-z)}\right)f'-\frac{1}{(z_0-z)^4}f=0.$$

Example $f_2(z)=1+4e^{\frac{1}{(z_0-z)}}+6e^{\frac{2}{(z_0-z)}}$ *is a solution of the differential equation*

$$f''+\left(\frac{1}{(z_0-z)^2}e^{\frac{-1}{(z_0-z)}}-\frac{2}{(z_0-z)}\right)f'-\frac{4}{(z_0-z)^4}f=0.$$

Theorem *Let* $A_0(z)\not\equiv 0, A_1(z),\dots,A_{k-1}(z)$ *be meromorphic functions in* $\overline{\mathbb{C}}-\{z_0\}$ *satisfying*

$$|A_0(z)|\geq \exp\left\{\frac{\alpha}{r^{\mu}}\right\},$$

$$|A_j(z)|\leq \exp\left\{\frac{\beta}{r^{\mu}}\right\},\ j\neq 0,$$

where $\alpha>\beta\geq 0,\ \mu>0,\ \arg(z_0-z)=\theta\in(\theta_1,\theta_2)\subset[0,2\pi)$ *and* $|z_0-z|=r\to 0$ *. Then, every solution* $f(z)\not\equiv 0$ *of the differential equation*

$$f^{(k)}+A_{k-1}(z)f^{(k-1)}+\dots+A_1(z)f'+A_0(z)f=0,$$

satisfies $\sigma_2(f,z_0)\geq\mu.$

Similar results to Theorem t3 in the complex plane are given in [bel, gund3].

Theorem *Let* $A_0(z)\not\equiv 0, A_1(z),\dots,A_{k-1}(z)$ *be analytic functions in* $\overline{\mathbb{C}}-\{z_0\}$ *satisfying* $\max\{\sigma(A_j,z_0):\ j\neq 0\}<\sigma(A_0,z_0)$ *. Then, every solution* $f(z)\not\equiv 0$ *of (eq3) satisfies* $\sigma_2(f,z_0)=\sigma(A_0,z_0).$

Preliminaries lemmas

Throughout this paper, we use the following notations that are not necessarily the same at each occurrence:

$r_0 > 0,\ \varepsilon > 0,\ \gamma > 1,\ \lambda > 0$ are real constants.

$E_1^* \subset (0, r_0]$ that has finite logarithmic measure $\int_0^{r_0} \frac{\chi_{E_1^*}}{t} dt < \infty$.

$E_2^* \subset [0, 2\pi)$ that has a linear measure zero $\int_0^{2\pi} \chi_{E_2^*} dt = 0.$

Lemma *[gund1] Let be a transcendental meromorphic function in , and let* $\gamma > 1$ $\varepsilon > 0$ *be given real constants; then*

i) there exists a set $E_1 \subset (1, \infty)$ *that has a finite logarithmic measure and a constant that depends only on such that for all* $R = |w|$ *satisfying* $R \notin E_1$ *, we have*

$$\left|\frac{g^{(k)}(w)}{g(w)}\right| \le \lambda [T(\gamma R, g) \log T(\gamma R, g)]^k;$$

ii) there exists a set that has a linear measure zero and a constant that depends only on such that for all $\theta \in [0, 2\pi) \setminus E_2$ *there exists a constant* $R_0 = R_0(\theta) > 0$ *such that for all satisfying* $\arg z \in [0, 2\pi) \setminus E_2$ *and* $r = |z| > R_0$ *, we have*

$$\left|\frac{g^{(k)}(w)}{g(w)}\right| \le \lambda [T(\gamma R, g) R^{\varepsilon} \log T(\gamma R, g)]^k.$$

Lemma *Let* f *be a non constant meromorphic function in* $\overline{\mathbb{C}} - \{z_0\}$ *and set* $g(w) = f\left(z_0 - \frac{1}{w}\right)$ *. Then,* $g(w)$ *is meromorphic in* $\mathbb{C}$ *and we have*

$$T(R, g) = T_{z_0}\left(\frac{1}{R}, f\right).$$

Remark *By Lemma lem2, if* f *is a non constant meromorphic function in* $\overline{\mathbb{C}} - \{z_0\}$ *and* $g(w) = f\left(z_0 - \frac{1}{w}\right)$ *then* $\sigma(f, z_0) = \sigma(g)$.

Lemma *Let f be a non constant meromorphic function in* $\overline{\mathbb{C}} - \{z_0\}$ *and let* $\gamma > 1,\ \varepsilon > 0$ *be given constants; then*

i) there exists a set $E_1^* \subset (0, r_0]$ *that has finite logarithmic measure* $\int_0^{r_0} \frac{\chi_{E_1^*}}{t} dt < \infty$ *and a constant* $\lambda > 0$ *that depends only on* γ *such that for all* $r = |z - z_0|$ *satisfying* $r \in (0, r_0] \setminus E_1^*$ *, we have*

$$\left|\frac{f^{(k)}(z)}{f(z)}\right| \leq \lambda\left[\frac{1}{r^2}T_{z_0}\left(\frac{r}{\gamma}, f\right)\log T_{z_0}\left(\frac{r}{\gamma}, f\right)\right]^k \quad (k \in \mathrm{N});$$

ii) there exists a set $E_2^* \subset [0, 2\pi)$ *that has a linear measure zero and a constant* $\lambda > 0$ *that depends only on* γ *such that for all* $\theta \in [0, 2\pi) \setminus E_2^*$ *there exists a constant*

such that for all satisfying $\arg(z - z_0) = \theta$ *and* $r = |z - z_0| < r_0$ *, we have*

$$\left|\frac{f^{(k)}(z)}{f(z)}\right| \leq \lambda\left[\frac{1}{r^{2+\varepsilon}}T_{z_0}\left(\frac{r}{\gamma}, f\right)\log T_{z_0}\left(\frac{r}{\gamma}, f\right)\right]^k \quad (k \in \mathrm{N}).$$

Lemma *Let* h *be a non constant analytic function in* $\overline{\mathrm{C}} - \{z_0\}$ *of order* $\sigma(f, z_0) > \alpha > 0$ *. Then, there exists a set* $F \subset (0, r_0]$ *of infinite logarirhmic measure* $\int_0^{r_0} \frac{\chi_F}{t} dt = \infty$ *such that for all* $r \in F$ *and* $|h(z)| = M_{z_0}(r, h)$*, we have*

$$\log|h(z)| > \frac{1}{r^\alpha}.$$

Lemma *[chian] Let* A_j *be meromorphic functions in* C *and* f *be a meromorphic solution of (eq3), assuming that not all coefficients are constants. Given a real constant , and denoting* $T(R) := \sum_{j=0}^{k-1} T(R, A_j)$ *, we have*

$$\log m(R, f) < T(R)\{\log R \log T(R)\}^\gamma.$$

We can transform this result near a singular point as the following.

Lemma *Let* A_j *be meromorphic functions in* $\overline{\mathrm{C}} - \{z_0\}$ *and* f *be a meromorphic solution of (eq3) in* $\overline{\mathrm{C}} - \{z_0\}$ *, assuming that not all coefficients* A_j *are constants. Given a real constant , and denoting* $T_{z_0}(r) := T_{z_0}(r, A_0) + \sum_{j=1}^{k-1}\sum_{i=j}^{k-1} T_{z_0}(r, A_i) + O(\log\frac{1}{r})$ *, we have*

$$\log m_{z_0}(r, f) < T_{z_0}(r)\left\{\log\frac{1}{r}\log(T_{z_0}(r))\right\}^\gamma.$$

Lemma *Let* $A(z)$ *be analytic function in* $\overline{\mathrm{C}} - \{z_0\}$ *with* $\sigma(A, z_0) < n$*. Set*

$g(z) = A(z)\exp\left\{\frac{a}{(z_0 - z)^n}\right\}$, *($n \geq 1$ is an integer)*

, $a = \alpha + i\beta \neq 0,\ z_0 - z = re^{i\varphi},\ \delta_a(\varphi) = \alpha\cos(n\varphi) + \beta\sin(n\varphi),$ *and*

$H = \{\varphi \in [0, 2\pi) : \delta_a(\varphi) = 0\}$, *(obviously, H is of linear measure zero). Then for any given $\varepsilon > 0$ and for any $\varphi \in [0, 2\pi) \setminus H$, there exists $r_0 > 0$ such that for we have*

(i) if $\delta_a(\varphi) > 0$, then

$$\exp\left\{(1-\varepsilon)\delta_a(\varphi)\frac{1}{r^n}\right\} \leq |g(z)| \leq \exp\left\{(1+\varepsilon)\delta_a(\varphi)\frac{1}{r^n}\right\},$$

(ii) if $\delta_a(\varphi) < 0$, then

$$\exp\left\{(1+\varepsilon)\delta_a(\varphi)\frac{1}{r^n}\right\} \leq |g(z)| \leq \exp\left\{(1-\varepsilon)\delta_a(\varphi)\frac{1}{r^n}\right\}.$$

Using (p8)-(p9) with (l3) in (p7), we get

$$\exp\left\{\frac{1}{r^{\alpha}}\right\} \leq \frac{\lambda}{r^{2k}}\left[T_{z_0}\left(\frac{r}{\gamma}, f\right)\right]^{2k} \exp\left\{\frac{1}{r^{\beta+\varepsilon}}\right\}.$$

REFERENCES

I. Amemiya and M. Ozawa, *Non-existence of finite order solutions of* $w'' + e^{-z}w' + Q(z)w = 0$, Hokkaido Math. J., **10** (1981), 1-17.

B. Belaidi and S. Hamouda, *Orders of solutions of an n-th order linear differential equation with entire coefficients*, Electron. J. Differential Equations, Vol. 2001(2001), No. 61, pp. 1-5.

L. Bieberbach, *Theorie der gewöhnlichen Differentialgleichungen*, Springer-Verlag, Berlin/Heidelberg/New York, 1965.

Z.X. Chen; *The growth of solutions of* $f'' + e^{-z}f' + Q(z)f = 0$, *where the order* $(Q) = 1$, Sci, China Ser. A, **45** (2002), 290-300.

Z.X. Chen and K. H. Shon; *On the growth of solutions of a class of higher order linear differential equations*, Acta. Mathematica Scientia, **24** B (1) (2004), 52-60.

Y.-M. Chiang and W.K. Hayman, *Estimates on the growth of meromorphic solutions of linear differential equations*, Comment. math. helv. 79 (2004) 451-470.

M. Frei, *Über die Subnormalen Lösungen der Differentialgleichung*

$w'' + e^{-z} w' + (Konst.) w = 0$, Comment. Math. Helv. 36 (1962), 1-8.

I. Chyzhykov, G. Gundersen and J. Heittokangas, *Linear differential equations and logarithmic derivative estimates*, Proc. London Math. Soc., 86 (2003), 735-754.

G. G. Gundersen, *Estimates for the logarithmic derivative of a meromorphic function, plus similar estimates,* J. Lond. Math. Soc. (2), 37 (1988), 88-104.

G. G. Gundersen, *On the question of whether* $f'' + e^{-z} f' + B(z) f = 0$ *can admit a solution* $f \not\equiv 0$ *of finite order*, Proc. Roy. Soc. Edinburgh **102A** (1986), 9-17.

G. Gundersen, *Finite order solutions of second order linear di erential equations*, Trans. Amer. Math. Soc. 305 (1988), pp. 415-429.

S. Hamouda, *Properties of solutions to linear differential equations with analytic coefficients in the unit disc,* Electron. J. Differential Equations, Vol 2012 (2012), No. 177, pp. 1-9.

S. Hamouda, *Iterated order of solutions of linear differential equations in the unit disc*, Comput. Methods Funct. Theory, 13 (2013) No. 4, 545-555.

W.K. Hayman, *Meromorphic functions*, Clarendon Press, Oxford, 1964.

J. Heittokangas, *On complex differential equations in the unit disc*, Ann. Acad. Sci. Fenn. Math. Diss. 122 (2000), 1-14.

J. Heittokangas, R. Korhonen and J. Rättyä, *Growth estimates for solutions of linear complex differential equations*, Ann. Acad. Sci. Fenn. Math. 29 (2004), No. 1, 233-246.

J. Heittokangas, R. Korhonen and J. Rättyä, *Fast growing solutions of linear differential equations in the unit disc*, Result. Math. 49 (2006), 265-278.

A.Ya. Khrystiyanyn, A.A. Kondratyuk, *On the Nevanlinna theory for meromorphic functions on annuli*, Matematychni Studii 23 (1) (2005) 19--30.

A.A. Kondratyuk, I. Laine, *Meromorphic functions in multiply connected domains,* in: Fourier Series Methods in Complex Analysis, in: Univ. Joensuu Dept. Math. Rep. Ser., vol. 10, Univ. Joensuu, Joensuu, 2006, pp. 9-111.

R. Korhonen, *Nevanlinna theory in an annulus,* in: Value Distribution Theory and Related Topics, in: Adv. Complex Anal. Appl., vol. 3, Kluwer Acad. Publ., Boston, MA, 2004, pp. 167-179.

I. Laine, *Nevanlinna theory and complex differential equations,* W. de Gruyter, Berlin, 1993.

E. L. Mark and Y. Zhuan, *Logarithmic derivatives in annulus,* J. Math. Anal. Appl. 356 (2009) 441-452.

L. Yang, *Value distribution theory,* Springer-Verlag Science Press, Berlin-Beijing. 1993.

MORGAN-VOYCE FONKSİYONLARINI KULLANARAK LİNEER LANE-EMDEN DİFERANSİYEL DENKLEMLERİNİN NÜMERİK ÇÖZÜMLERİ VE REZİDÜEL DÜZELTME

NUMERICAL SOLUTIONS OF THE LINEAR LANE-EMDEN DIFFERENTIAL EQUATIONS BY USING THE MORGAN-VOYCE FUNCTIONS AND RESIDUAL CORRECTIONS

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ÖZET

Bu çalışmanın amacı, lineer Lane-Emden diferansiyel denklemlerinin yaklaşık çözümlerini, Morgan-Voyce polinomları yardımıyla bulmaktır. Yöntemde, yaklaşık çözümlerin matris temsili ve yaklaşık çözümlerin türevlerinin matris temsili Morgan-Voyce polinomlarına bağlı olacak şekilde belirlenir. Bu yönteme göre verilen problem Morgan-Voyce katsayılarını içeren bir cebirsel sisteme indirgenmiştir. Morgan-Voyce katsayıları bu sistem çözülerek elde edilir. Bu katsayılar çözüm formuna yazılır ve yaklaşık çözüm Morgan-Voyce polinomlarına bağlı olarak bulunur. Ek olarak, rezidüel hata fonksiyonu ile bir hata problemi oluşturulur ve bu hata problemi Morgan-Voyce kollokasyon metodu kullanılarak çözülür. Bu yönteme göre, problemin tam çözümü bilinmediğinde, hatalar yaklaşık olarak hesaplanabilir. Bu iki yöntem iki örnek için uygulanır. Sonuçlar tablolar ve grafiklerde gösterilir. Bu sonuçlara göre, literatürdeki diğer yöntemlere göre daha iyi sonuç verdiği gözlemlenir. Dolayısıyla tüm bunlardan yöntemin başarılı olduğu söylenebilir. Ayrıca, sonuçlar Matlab programında yazılan kodlar ile elde edilir.

Anahtar Kelimeler: Kollokasyon Noktaları, Kollokasyon Yöntemi, Lane–Emden Diferansiyel Denklemleri, Morgan-Vorgan polinomları

ABSTRACT

The aim of this study is to find approximate solutions of linear Lane-Emden differential equations with the help of Morgan-Voyce polynomials. In the method, the matrix representation of the approximate solutions and the matrix representation of the derivatives of the approximate solutions are determined depending on Morgan-Voyce polynomials. The problem given according to this method is reduced to an algebraic system containing Morgan-Voyce coefficients. Morgan-Voyce coefficients are obtained by solving this system. These coefficients are written in the solution form and the approximate solution is found based on Morgan-Voyce polynomials. In addition, an error problem is created with the residual error function and this error problem is solved using the Morgan-Voyce collocation method. According to this method, the errors can be approximated when the exact solution to the problem is unknown. These two methods are applied for two examples. Results are shown in tables and graphs. According to these results, it is observed that it gives better results than other methods in the literature. Therefore, it can be said that the method is successful from all these. Also, the results are obtained with the codes written in the Matlab program.

Keywords: Collocation Method, Collocation Points, Lane–Emden Differential Equations, Morgan-Vorgan Polynomials

1. INTRODUCTION

Lane-Emden type equations can be modeled many phenomena such as thermal explosions (Chambre, 1952), stellar structure (Chandrasekhar, 1967) and the thermal behavior of a spherical gas cloud, isothermal gas spheres and thermionic currents (Richardson, 1921). There are many analytical methods for Lane-Emden type equations. However, it is difficult or impossible to solve analytically. Therefore, numerical methods such as the Legendre wavelets (Yousefi, 2006), the Bessel collocation method (Yüzbaşı & Sezer, 2011), (Yüzbaşı & Sezer, 2013) the Hermite functions collocation method (Parand, Dehghan, Rezaei, & Ghader, 2010), the variational iteration method (Yildirim & Öziş, 2009), (Dehghan & Shakeri, 2008), the B-spline method (Caglar & Caglar, 2006), the homotopy perturbation method (Yildirim & Öziş, 2007), (Ramos, 2008), (Chowdhury & Hashim, 2009), the rational Legendre pseudospectral method (Parand, Shahini, & Dehghan, 2009), the Adomian decomposition method (Wazwaz, 2001), the Pade series method (Vanani & Aminataei, 2010), the nonperturbative approximate method (Shawagfeh, 1993), and the variational approach method (He, 2003), have been studied. On the other hand, numerical solutions of high-order linear differential-difference equations (Türkyılmaz, Gürbüz, & Sezer, 2016), generalized functional integro-differential equations of Volterra-Type (Ozel, Kürkçü, & Sezer, 2019) and nonlinear ordinary differential equations with quadratic and cubic terms (Tarakçı, Özel, & Sezer, 2020) related to Morgan-Voyce polynomials have been used to solve numerically.

In this paper, we will study the Lane - Emden type differential equations

$$L[y(x)] = y''(x) + \frac{\alpha}{x} y'(x) + p(x)y(x) = g(x), \qquad 0 \le x \le b \tag{1}$$

with the conditions

$$\sum_{k=0}^{1}(a_{jk}y^{(k)}(0)+b_{jk}y^{(k)}(b))=\lambda_j, \qquad j=0,1. \tag{2}$$

Here, while $p(x)$ and $g(x)$ are the defined functions on interval $0\leq x\leq b$, α, a_{jk}, b_{jk} and λ_j are real or complex constants. Also, $y^{(0)}(x)=y(x)$ is the function to be determined.

The first aim of this study is to obtain the approximate solution of the Equation (1)-(2) depending on the Morgan-Voyce polynomials in the form

$$y_N(x)=\sum_{n=0}^{N}a_n\,B_n(x) \tag{3}$$

The second aim of this study is to obtain better approximate solution by using the residual error estimation technique as

$$y_{N,M}(x)=y_N(x)+e_{N,M}(x) \tag{4}$$

where

$$e_{N,M}(x)=\sum_{n=0}^{M}a_n^*\,B_n(x). \tag{5}$$

In the Equations (3)-(5), N and M are any chosen positive integers such that $M\geq N\geq 2$ and a_n, a_n^* are the unknown Morgan-Voyce coefficients. Also, $B_n(x)$ are the Morgan-Voyce polynomials defined by (Stoll & Tichy, 2008)

$$B_n(x)=\sum_{k=0}^{n}\binom{n+k+1}{n-k}x^k. \tag{6}$$

The recurrence relationship of Morgan-Voyce polynomials $B_n(x)$ is (Stoll & Tichy, 2008)

$$B_n(x)=(x+2)B_{n-1}(x)-B_{n-2}(x), n\geq 2$$

such that the first two Morgan-Voyce polynomials are $B_0(x)=1$ and $B_1(x)=x+2$.

Additionally, the Morgan-Voyce polynomials $B_n(x)$ are solutions of the differential equation (Stoll & Tichy, 2008)

$$x(x+4)B_n''(x)+3(x+2)B_n'(x)-n(n+2)B_n(x)=0$$

2. MORGAN-VOYCE COLLOCATION METHOD

In this section, we write the matrix form of the approximate solution of (3) as

$$y_N(x) = \mathbf{B}(x)\mathbf{A} \tag{7}$$

where $\mathbf{B}(x) = [B_0(x) \quad B_1(x) \quad \cdots \quad B_N(x)]$ and $\mathbf{A} = [a_0 \quad a_1 \quad \cdots \quad a_N]^T$. After, through the relations (6), we write the matrix $\mathbf{B}(x)$ in (7) as

$$\mathbf{B}(x) = \mathbf{X}(x)\mathbf{D} \tag{8}$$

where

$$\mathbf{X}(x) = [1 \quad x \quad \cdots \quad x^N] \text{ and } \mathbf{D}^T = \begin{bmatrix} \binom{1}{0} & 0 & \cdots & 0 \\ \binom{2}{1} & \binom{3}{0} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \binom{N+1}{N} & \binom{N+2}{N-1} & \cdots & \binom{2N+1}{0} \end{bmatrix}$$

So, we take the first derivative of the matrix form (8) in the form

$$\mathbf{B}'(x) = \boldsymbol{X}'(x)\boldsymbol{D} = \boldsymbol{X}(x)\mathbf{M}\boldsymbol{D} \tag{9}$$

and we take the second derivative of the matrix form (8) in the form

$$\mathbf{B}''(x) = \boldsymbol{X}''(x)\boldsymbol{D} = \boldsymbol{X}(x)\mathbf{M}^2\boldsymbol{D} \tag{10}$$

where

$$\mathbf{M}^T = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 2 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & N-1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & N & 0 \end{bmatrix}.$$

Thus, by using the matrix form (9) and (10), we can write the first derivative of the matrix form (7)

$$y_N'(x) = \mathbf{B}'(x)\mathbf{A} = \boldsymbol{X}(x)\mathbf{M}\boldsymbol{D} \tag{11}$$

and we can write the second derivative of the matrix form (7)

$$y_N''(x) = \mathbf{B}''(x)\mathbf{A} = \boldsymbol{X}(x)\mathbf{M}^2\boldsymbol{D}\boldsymbol{A} \tag{12}$$

Thirdly, by substituting the matrix relations (7), (8), (11) and (12) in the Equation (1), we get the matrix equation

$$\boldsymbol{X}(x)\mathbf{M}^2\boldsymbol{D}\mathbf{A}+\frac{\alpha}{x}\boldsymbol{X}(x)\mathbf{M}\boldsymbol{D}+p(x)\mathbf{X}(x)\mathbf{DA}=\mathbf{g}(x). \tag{13}$$

Next, we define the collocation points as

$$x_i=a+\frac{b-a}{N}i, \qquad i=0,1,\dots,N \tag{14}$$

and by writing these collocation points (14) instead of x in the Equation (13), we obtain as follows:

$$\boldsymbol{X}(x_i)\mathbf{M}^2\boldsymbol{D}\mathbf{A}+\frac{\alpha}{x_i}\boldsymbol{X}(x_i)\mathbf{M}\boldsymbol{D}+p(x_i)\mathbf{X}(x_i)\mathbf{DA}=\mathbf{g}(x_i) \tag{15}$$

or

$$\{\boldsymbol{X}\mathbf{M}^2\boldsymbol{D}+\boldsymbol{\alpha}\boldsymbol{X}\mathbf{M}\boldsymbol{D}+\mathbf{PXD}\}\mathbf{A}=\mathbf{G} \tag{16}$$

where

$$\boldsymbol{X}=[\boldsymbol{X}(x_0)\quad \boldsymbol{X}(x_1)\quad \cdots \quad \boldsymbol{X}(x_N)]^{\boldsymbol{T}},\ \mathbf{G}=[g(x_0)\quad g(x_1)\quad \cdots \quad g(x_N)]^{\boldsymbol{T}},$$

$$\boldsymbol{\alpha}=\begin{bmatrix}\frac{\alpha}{x_0} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \frac{\alpha}{x_1} & \cdots & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \mathbf{0} & \cdots & \frac{\alpha}{x_N}\end{bmatrix},\ \mathbf{P}=\begin{bmatrix}p(x_0) & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & p(x_1) & \cdots & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \mathbf{0} & \cdots & p(x_N)\end{bmatrix}.$$

From here, we can write the matrix equation (16) also as

$$\mathbf{WA}=\mathbf{G},\qquad \mathbf{W}=\boldsymbol{X}\mathbf{M}^2\boldsymbol{D}+\boldsymbol{\alpha}\boldsymbol{X}\mathbf{M}\boldsymbol{D}+\mathbf{PXD};\quad \text{or}\quad [\mathbf{W};\mathbf{G}]. \tag{17}$$

Then, we write the collocation points (14) in the conditions (2) with the help of (7)-(8), and we obtain the matrix equation corresponding to the conditions in the form

$$\sum_{k=0}^{1}\left[a_{jk}\boldsymbol{X}(0)\mathbf{M}^{k}\boldsymbol{D}+b_{jk}\boldsymbol{X}(b)\mathbf{M}^{k}\boldsymbol{D}\right]\mathbf{A}=[\lambda_j], \qquad j=0,1 \tag{18}$$

or

$$\mathbf{U}_j\mathbf{A}=[\lambda_j],\ \mathbf{U}_j=\sum_{k=0}^{1}\left[a_{jk}\boldsymbol{X}(0)\mathbf{M}^{k}\boldsymbol{D}+b_{jk}\boldsymbol{X}(b)\mathbf{M}^{k}\boldsymbol{D}\right];\quad or\quad [\mathbf{U}_j;\lambda_j]. \tag{19}$$

So, the number of rows of the fundamental matrix system (17) is $N+1$ and the number of rows of the matrix form for the conditions (19) is 2. Hence, we write the $N-1$ rows of the system (17) and the rows of the conditions (19) in a single matrix in the form

$$[\widetilde{\mathbf{W}};\widetilde{\mathbf{G}}]=\begin{bmatrix} w_{0,0} & w_{0,1} & w_{0,2} & \cdots & w_{0,N} & ; & g(x_0) \\ w_{1,0} & w_{1,1} & w_{1,2} & \cdots & w_{1,N} & ; & g(x_1) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ w_{N-2,0} & w_{N-2,1} & w_{N-2,2} & \cdots & w_{N-2,N} & ; & g(x_{N-2}) \\ u_{0,0} & u_{0,1} & u_{0,2} & \cdots & u_{0,N} & ; & \lambda_0 \\ u_{1,0} & u_{1,1} & u_{1,2} & \cdots & u_{1,N} & ; & \lambda_1 \end{bmatrix} \tag{20}$$

Consequently, if $rank\ \widetilde{\mathbf{W}}=rank[\widetilde{\mathbf{W}};\widetilde{\mathbf{G}}]=N+1$, it can be written

$$\mathbf{A}=(\widetilde{\mathbf{W}})^{-1}\widetilde{\mathbf{G}}. \tag{21}$$

and by solving this system, the coefficients matrix $\mathbf{A}$ is found depending on Morgan-Voyce polynomials.Then, by substituting this matrix $\mathbf{A}$ in the Equation (3), we determine Morgan-Voyce polynomial solutions. Additionally, when, $|\widetilde{\mathbf{W}}|=0$, if $rank\ \widetilde{\mathbf{W}}=rank[\widetilde{\mathbf{W}};\widetilde{\mathbf{G}}]<N+1$, then we may find a particular solution. Otherwise if $rank\ \widetilde{\mathbf{W}}\neq rank[\widetilde{\mathbf{W}};\widetilde{\mathbf{G}}]$, then we can't find a solution.

3. RESIDUAL CORRECTION AND ERROR ESTIMATION

In this section, we will construct an error estimation method which depends on the residual error function. Also, we will develop Morgan-Voyce polynomial solutions with the help of this function. With $y_N(x)$ is the Morgan-Voyce polynomial solution of the problem (1)-(2), let's deal with the residual function as

$$R_N(x)=L[y_N(x)]-g(x) \tag{22}$$

Then, we can write

$$\begin{cases} L[y_N(x)]=y_N^{(2)}(x)+\dfrac{\alpha}{x}y_N^{(1)}(x)+p(x)y_N(x)=g(x)+R_N(x), \\ \displaystyle\sum_{k=0}^{1}\left(a_{jk}y_N^{(k)}(0)+b_{jk}y_N^{(k)}(b)\right)=\lambda_j, \qquad j=0,1. \end{cases} \tag{23}$$

Thus, we can constitute the error problem with the help of (1)-(2)-(22) as

$$\begin{cases} L[e_N(x)] = L[y(x)] - L[y_N(x)] = -R_N(x) \\ \sum_{k=0}^{1}\left(a_{jk}e_N^{(k)}(0) + b_{jk}e_N^{(k)}(b)\right) = 0, \qquad j = 0,1. \end{cases} \tag{24}$$

or

$$\begin{cases} e_N^{(2)}(x) + \frac{\alpha}{x}e_N^{(1)}(x) + p(x)e_N(x) = -R_N(x), \\ \sum_{k=0}^{1}\left(a_{jk}e_N^{(k)}(0) + b_{jk}e_N^{(k)}(b)\right) = 0, \qquad j = 0,1. \end{cases} \tag{25}$$

Here, we can define the actual error function $e_N(x)$ in the form

$$e_N(x) = y(x) - y_N(x). \tag{26}$$

Hence, $e_{N,M}(x)$ is the estimated error function and we use this function when the exact solution of the problem (1)-(2) is unknown. On the other hand, we solve the problem (24) by using the method in Section 2 in the form

$$e_{N,M}(x) = \sum_{n=0}^{M} a_n^* \, B_n(x). \tag{27}$$

Also, we define the improved approximate solution as $y_{N,M}(x) = y_N(x) + e_{N,M}(x)$ and we calculate the error of this improved approximate solution as

$$E_{N,M}(x) = |y(x) - y_{N,M}(x)|. \tag{28}$$

4. APPLICATIONS OF THE METHOD

In this section, we will apply the methods discussed in Sections 2 and 3 on 2 examples and show the results in tables and graphs. We will also make comparisons with the results of other methods. Matlab is used for all calculations.

Example 4.1. Firstly, we deal with the Lane-Emden problem

$$\begin{cases} y''(x) + \frac{1}{x}y'(x) = \left(\frac{8}{8-x^2}\right)^2, 0 \le x \le 1 \\ \quad y(1) = 0, \qquad y'(0) = 0 \end{cases} \tag{29}$$

Here, the exact solution of the problem (29) is $y(x) = 2\,log\left(\frac{7}{8-x^2}\right)$, $\alpha = 1$, $P = 0$ and $g(x) = \left(\frac{8}{8-x^2}\right)^2$. By applying the method in Section 2 for N = 4, we write the approximate solution as

$$y_4(x) = \sum_{n=0}^{4} a_n\, B_n(x) \tag{30}$$

and we calculate the set of collocation points (14) for $a = 0.001$ and $b = 1$ as

$$\left\{x_0 = \frac{1}{1000}, x_1 = \frac{418}{1667}, x_2 = \frac{501}{1001}, x_3 = \frac{751}{1001},\ x_4 = 1\right\} \tag{31}$$

Hence, the fundamental matrix equation is

$$\mathbf{WA} = \mathbf{G}. \tag{32}$$

or

$$\mathbf{W} = \begin{bmatrix} 0 & 1000 & 4004 & 10024009/1000 & 1255254501/62500 \\ 0 & 4000/1003 & 20012/1003 & 265342081/4012000 & 183369189027/1003000000 \\ 0 & 2000/1001 & 12004/1001 & 97066009/2002000 & 41042012001/250250000 \\ 0 & 4000/3001 & 28004/3001 & 529150009/12004000 & 521219027001/3001000000 \\ 0 & 1 & 8 & 43 & 192 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 1 \\ 1469/1446 \\ 146/137 \\ 1451/1254 \\ 64/49 \end{bmatrix} \tag{33}$$

and for conditions, we have

$$\mathbf{U} = \begin{bmatrix} 1 & 3 & 8 & 21 & 55 \\ 0 & 1 & 4 & 10 & 20 \end{bmatrix}, \boldsymbol{\lambda} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{34}$$

Then, we write the second row of the matrix **W** and the matrix **U** in a single matrix as

$$\tilde{\mathbf{W}} = \begin{bmatrix} 0 & 1000 & 4004 & 10024009/1000 & 1255254501/62500 \\ 0 & 4000/1003 & 20012/1003 & 265342081/4012000 & 183369189027/1003000000 \\ 0 & 2000/1001 & 12004/1001 & 97066009/2002000 & 41042012001/250250000 \\ 1 & 3 & 8 & 21 & 55 \\ 0 & 1 & 4 & 10 & 20 \end{bmatrix}, \tilde{\mathbf{G}} = \begin{bmatrix} 1 \\ 1469/1446 \\ 146/137 \\ 0 \\ 0 \end{bmatrix} \tag{35}$$

Hence, we solve the system $[\widetilde{\mathbf{W}}; \widetilde{\mathbf{G}}]$ in Matlab and so we obtain the matrix of coefficients **A** as

$$\mathbf{A} = \begin{bmatrix} 1392/817 & -973/534 & 1435/2017 & -146/1071 & 100/5891 \end{bmatrix}^{\mathrm{T}}. \tag{36}$$

Thus, the Morgan-Voyce polynomial solution is

$$\begin{aligned} y_4(x) = 0.01697503613758x^4 - 5.209000391975185e-04x^3 \\ + 0.250001166624955x^2 - 0.266455302723344 \end{aligned} \tag{37}$$

And the actual error function becomes

$$\begin{aligned} e_4(x) = 0.00950388237888242x^3 + 0.249978678764659x^2 \\ + 4.33680868994202e - 19x - 0.259482561143542 \\ - 2\,log\left(\frac{7}{8 - x^2}\right). \end{aligned} \tag{38}$$

Now, we constitute the error problem

$$\begin{cases} e_4^{(2)}(x) + \dfrac{1}{x} e_4^{(1)}(x) = -R_4(x), \\ e_4^{(1)}(0) = 0, e_4(1) = 0 \end{cases} \tag{39}$$

where

$$R_4(x) = y_4^{(2)}(x) + \frac{1}{x} y_4^{(1)}(x) - \left(\frac{8}{8 - x^2}\right)^2. \tag{40}$$

Then, by applying the method in Section 3, we solve the error problem (39) for $M = 6$ and so we can obtain the estimated error function, the improved approximate solution and the improved error function. The exact solution of the system (29), Bessel polynomial solution, Morgan-Voyce polynomial solution, improved Bessel polynomial solution and improved Morgan-Voyce polynomial solution are given in Table 1. The actual error function of the system (29), the estimated error function and the improved error function are given for $N = 7$ and $M = 10$ in Table 2 and Figure 1 at different points for the Bessel and Morgan-Voyce methods. In Table 3, the actual error functions of the system (29) are given for different N values. From all these results, it can be said that the higher the value N, the better results are obtained and the method is very effective. In addition, although there is not much difference between the actual error results compared to the comparison made with the Bessel method (Yüzbaşı & Sezer, 2013), when we look at the improved error results, it is seen that the presented method is more effective.

Tablo 1 The exact solution, the approximate solution and the improved approximate solution of the system (29)

x_i	**Exact Solution**	**Bessel polynomial solution (Yüzbaşı & Sezer, 2013)**	**Morgan-Voyce polynomial solution**
	$y(x_i)$	$y_7(x_i)$	$y_7(x_i)$
0	−0.26706278524904525	−0.26706118400745205	-0.26705976695773307
0.2	-0.25703770160195668	−0.25703613506200951	-0.25703467401943167

0.4	-0.22665737061400635	−0.22665581293982964	-0.22665434134952904
0.6	-0.17497490824623164	−0.17497335588303395	-0.17497187877953834
0.8	-0.10029956737094313	−0.10029801545556499	-0.10029654003433037
1	0	−0.16197980456933e−15	0
	Exact Solution	**Improved Bessel polynomial solution** (Yüzbaşı & Sezer, 2013)	**Improved Morgan-Voyce polynomial solution**
	$y(x_i)$	$y_{7,10}(x_i)$	$y_{7,10}(x_i)$
0	−0.26706278524904525	−0.26706277509905801	-0.26706277509905738
0.2	-0.25703770160195668	−0.25703769146757538	-0.25703769146757476
0.4	-0.22665737061400635	−0.22665736048447666	-0.22665736048447604
0.6	-0.17497490824623164	−0.17497489811938738	-0.17497489811938671
0.8	-0.10029956737094313	−0.10029955723627511	-0.1002995572362745
1	0	−0.14493072540699e−15	0

Tablo 2 The actual errors, the estimated errors and improved errors of the system (29)

x_i	**The actual errors for PM**	**The estimated errors for PM**	**The improved errors for PM**
	$e_7(x_i)$	$e_{7,10}(x_i)$	$E_{7,10}(x_i)$
0	3.0183e-06	3.0081e-06	1.0150e-08
0.2	3.0276e-06	3.0174e-06	1.0134e-08
0.4	3.0293e-06	3.0191e-06	1.0130e-08
0.6	3.0295e-06	3.0193e-06	1.0127e-08
0.8	3.0273e-06	3.0172e-06	1.0135e-08

1	0	0	0
	The actual errors for (Yüzbaşı & Sezer, 2013)	**The estimated errors for** (Yüzbaşı & Sezer, 2013)	**The improved errors for** (Yüzbaşı & Sezer, 2013)
	$e_7(x_i)$	$e_{7,10}(x_i)$	$E_{7,10}(x_i)$
0	1.6012e−006	1.5911e−006	1.0150e−008
0.2	1.5665e−006	1.5564e−006	1.0134e−008
0.4	1.5577e−006	1.5475e−006	1.0130e−008
0.6	1.5524e−006	1.5422e−006	1.0127e−008
0.8	1.5519e−006	1.5418e−006	1.0135e−008
1	1.6198e−016	8.3009e−020	1.4494e−016

Tablo 3 The actual errors of the system (29)

x_i	**The actual errors for PM**	**The actual errors for PM**	**The actual errors for PM**
	$e_4(x_i)$	$e_7(x_i)$	$e_{12}(x_i)$
0	6.0748e-04	3.0183e-06	2.5208e-10
0.2	6.0544e-04	3.0276e-06	2.5178e-10
0.4	6.0348e-04	3.0293e-06	2.5168e-10
0.6	6.0748e-04	3.0295e-06	2.5162e-10
0.8	5.3129e-04	3.0273e-06	2.5158e-10
1	0	0	0

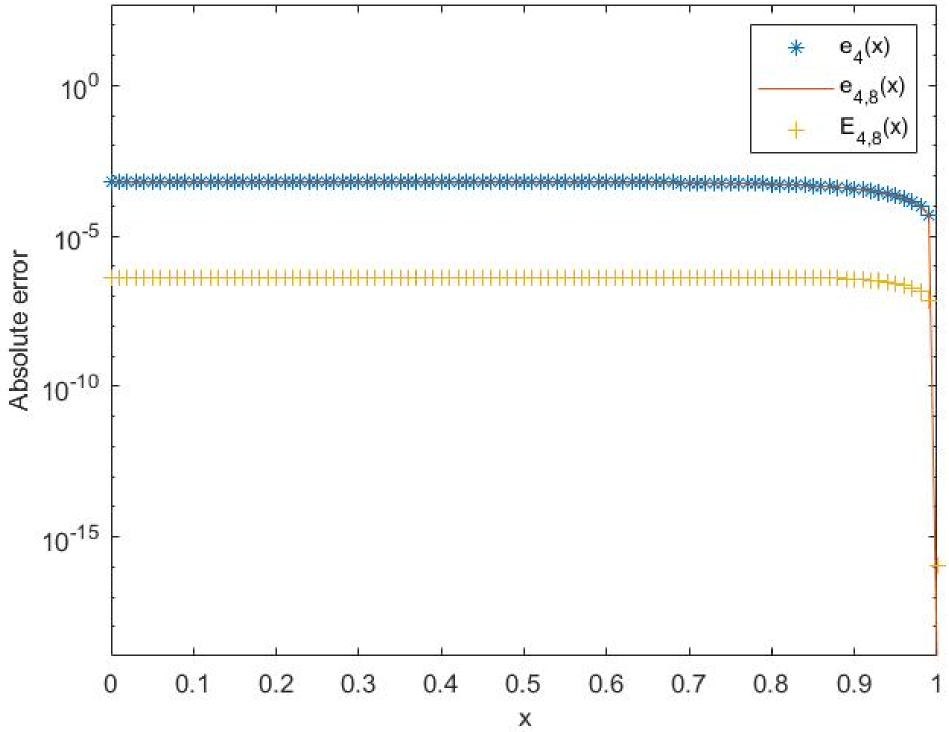

Figure 1 The actual error, the estimated error and improved error of the system (29)

Example 4.2. Secondly, we deal with the Lane-Emden problem

$$\begin{cases} y''(x) + \dfrac{2}{x} y'(x) + y(x) = 6 + 12x + x^2 + x^3 \\ \qquad y(0) = 0, \qquad y'(0) = 0 \end{cases}, 0 \le x \le 1 \qquad (41)$$

Here, the exact solution of the problem (29) is $y(x) = x^2 + x^3$, $\alpha = 2$, $P = 1$ and $g(x) = 6 + 12x + x^2 + x^3$. The actual error, the estimated error and the improved error of the system (41) are given for $N = 5$ and $M = 8$ in Table 4 and Figure 2. From all these results, it can be said that the estimated errors very close to actual errors and improved errors give better results than actual errors and according to the method discussed in Section 3 the method is effective.

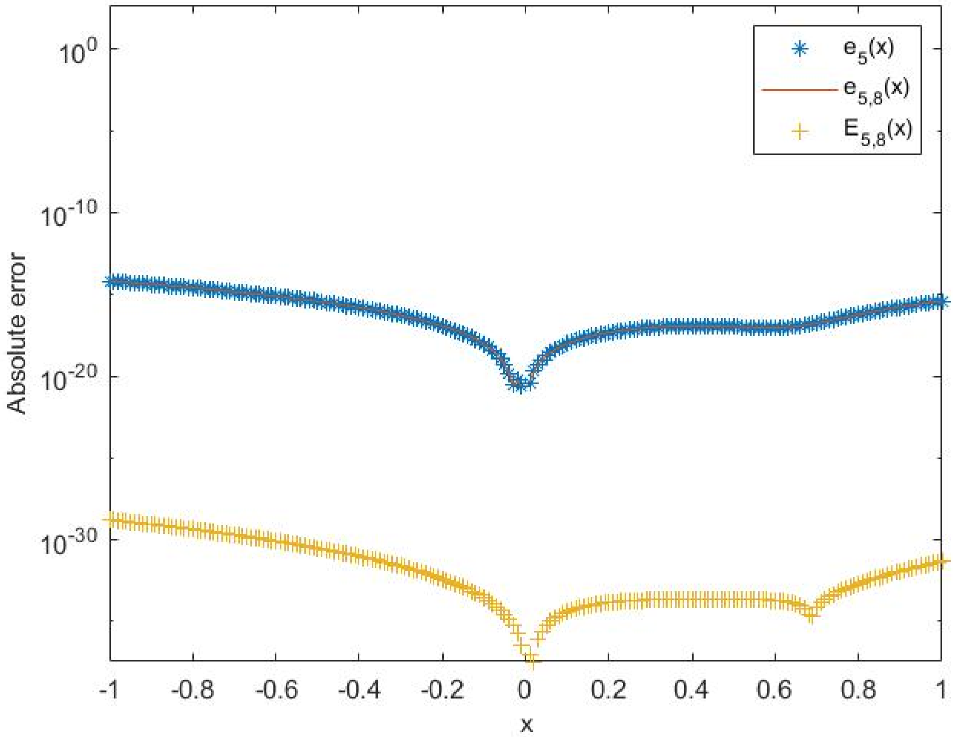

Figure 2 The absolute error of the system (41)

Tablo 4 The actual errors, the estimated errors and improved errors of the system (41)

x_i	**The actual errors for PM**	**The estimated errors for PM**	**The improved errors for PM**
	$e_5(x_i)$	$e_{5,8}(x_i)$	$E_{5,8}(x_i)$
0	0	0	0
0.2	4.58717222697e-18	4.58717222697e-18	1.45212328405e-34
0.4	1.15388333652e-17	1.15388333652e-17	2.372568381e-34
0.6	8.97752291734e-18	8.97752291734e-18	1.91800258528e-34
0.8	5.9154975339e-17	5.9154975339e-17	1.96795990081e-33
1	3.9388474237e-16	3.9388474237e-16	4.91666451354e-32

5. CONCLUSIONS

In this study, the Morgan-Voyce collocation method was applied to numerically solve the linear Lane-Emden equations. For this purpose, the problem (1) - (2) is transformed into an algebraic equation system containing the unknown coefficients of Morgan-Voyce series. By solving this system, Morgan-Voyce coefficients were determined and thus approximate solutions were obtained according to Morgan-Voyce polynomials. The method is applied in Section 4. Thus, it is understood whether the presented method is effective or not. When the results are examined, it can be said that the method is very effective. After making the necessary arrangements, the method can be developed for the nonlinear Lane – Emden equations.

REFERENCES

Caglar, N., & Caglar, H. (2006). B-spline solution of singular boundary value problems. *Appl. Math. Comput., 182*, 1509–1513.

Chambre, P. (1952). On the solution of the Poisson–Boltzmann equation with application to the theory of thermal explosions. *J. Chem. Phys., 20*, 1795–1797.

Chandrasekhar, S. (1967). *Introduction to the Study of Stellar Structure.* New York: Dover.

Chowdhury, M., & Hashim, I. (2009). Solutions of Emden–Fowler equations by homotopy-perturbation method. *Nonlinear Anal. RWA, 10*, 104–115.

Dehghan, M., & Shakeri, F. (2008). Approximate solution of a differential equation arising in astrophysics using the variational iteration method. *New Astron., 13*, 53–59.

He, J.-H. (2003). Variational approach to the Lane–Emden equation. *Appl. Math. Comput., 143*, 539–541.

Ozel, M., Kürkçü, Ö., & Sezer, M. (2019). Morgan-Voyce Matrix Method for Generalized Functional Integro-Differential Equations of Volterra-Type. *Journal of Science and Arts, 2*(47), 295-310.

Parand, K., Dehghan, M., Rezaei, A., & Ghader, S. (2010). An approximation algorithm for the solution of the nonlinear Lane–Emden type equations arising in astrophysics using Hermite functions collocation method. *Comput. Phys. Comm., 181*, 1096–1108.

Parand, K., Shahini, A., & Dehghan, M. (2009). Rational Legendre pseudospectral approach for solving nonlinear differential equations of Lane–Emden type. *J. Comput. Phys., 228*, 8830–8840.

Ramos, J. (2008). Series approach to the Lane–Emden equation and comparison with the homotopy perturbation method. *Chaos Soliton Fractals, 38*, 400–408.

Richardson, O. (1921). *The Emission of Electricity from Hot Bodies.* London, New York: Longman, Green and Co.

Shawagfeh, N. (1993). Nonperturbative approximate solution for Lane–Emden equation. *J. Math. Phys., 34*(9), 43–64.

Stoll, T., & Tichy, R. (2008). Diophantine equations for Morgan-Voyce and other modified orthogonal polynomials. *Mathematica Slovaca, 58*(1), 11-18.

Tarakçı, M., Özel, M., & Sezer, M. (2020). Solution of nonlinear ordinary differential equations with quadratic and cubic terms by Morgan-Voyce matrix-collocation method. *Turkish Journal of Mathematics*, *44*, 906-918.

Türkyılmaz, B., Gürbüz, B., & Sezer, M. (2016). Morgan-Voyce Polynomial Approach for Solution of High-Order Linear Differential-Difference Equations with Residual Error Estimation. *Düzce University Journal of Science & Technology*, *4*, 252-263.

Vanani, S., & Aminataei, A. (2010). On the numerical solution of differential equations of Lane–Emden type. *Comput. Math. Appl., 59*, 2815–2820.

Wazwaz, A. (2001). A new algorithm for solving differential equations of Lane–Emden type. *Appl. Math. Comput., 118*, 287–310.

Yildirim, A., & Öziş, T. (2007). Solutions of singular IVPs of Lane–Emden type by homotopy perturbation method. *Phys. Lett. A, 369*, 70–76.

Yildirim, A., & Öziş, T. (2009). Solutions of singular IVPs of Lane–Emden type by the variational iteration method. *Nonlinear Anal., 70*, 2480–2484.

Yousefi, S. (2006). Legendre wavelets method for solving differential equations of Lane–Emden type. *Appl. Math. Comput., 181*, 1417–1422.

Yüzbaşı, Ş., & Sezer, M. (2011). A collocation approach to solve a class of Lane–Emden type equations. *J. Adv. Res. Appl. Math., 3*(2), 58–73.

Yüzbaşı, Ş., & Sezer, M. (2013). An improved Bessel collocation method with a residual error function to solve a class of Lane–Emden differential equations. *Mathematical and Computer Modelling, 57*, 1298-1311.

PELL-LUCAS FONKSİYONLARINI KULLANARAK YÜKSEK MERTEBEDEN LİNEER FREDHOLM İNTEGRO-DİFERANSİYEL DENKLEM SİSTEMLERİ İÇİN OPERASYONEL BİR MATRİS YÖNTEMİ

AN OPERATIONAL MATRIX METHOD FOR THE SYSTEMS OF HIGHER-ORDER LINEAR FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS BY USING THE PELL-LUCAS FUNCTIONS

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ÖZET

Bu çalışmada, lineer Fredholm integro-diferansiyel denklem sistemlerini sayısal olarak çözmek için etkili bir algoritma kurulacaktır. Lineer Fredholm integro-diferansiyel denklem sistemlerinde çözümleri sayısal olarak ararız çünkü bazen tam çözüm yoktur veya tam çözümü elde etmek zordur. Bu çalışmanın amacı, lineer Fredholm integro-diferansiyel denklem sistemlerinin Pell-Lucas polinomlarına bağlı olarak yaklaşık çözümlerini bulmaktır. Yöntem, yaklaşık çözümün matris temsillerini, bu yaklaşık çözümlerin türevlerinin matris temsillerini ve bu yaklaşık çözümlerin integrallerinin matris temsillerini içerir, ki bu yaklaşık çözümler de Pell-Lucas serilerine bağlıdır. Bu yönteme göre verilen problem, Pell-Lucas katsayılarını içeren cebirsel bir sisteme indirgenir. Bu sistem çözülerek Pell-Lucas katsayıları elde edilir. Bu katsayılar çözüm formunda yazılır ve yaklaşık çözüm Pell-Lucas polinomlarına bağlı olarak bulunur. Son olarak yöntem uygulanır. Sonuçlar tablolar ve grafiklerde gösterilir. Buna göre, mevcut yöntemle elde edilen yaklaşık çözüm, tam çözüme çok yakındır. Sonuçlar incelenerek yöntemin başarılı olduğu söylenebilir. Ayrıca, sonuçlar Matlab programında yazılan kodlar ile elde edilmiştir.

Anahtar Kelimeler: Fredholm İntegro-Diferansiyel Denklemleri, Kollokasyon Noktaları, Kollokasyon Yöntemi, Pell-Lucas Polinomları

ABSTRACT

In this study, an effective algorithm will be established to numerically solve the systems of linear Fredholm integro-differential equations. In the systems of linear Fredholm integro-differential equation, we look for solutions numerically because sometimes there is no exact solution or it is difficult to get the exact solution. The aim of this study is to find the approximate solutions of the systems of linear Fredholm integro-differential equation depending on the Pell-Lucas polynomials. The method includes the matrix representations of approximate solution, the matrix representations of the derivatives and the matrix representations of the integrals of this approximate solution, which depend on the Pell-Lucas series. According to this method, the given problem is reduced to an algebraic system containing Pell-Lucas coefficients. Pell-Lucas coefficients are obtained, by solving this system. This coefficients are written in the solution form and the approximate solution is found depending on the Pell-Lucas polynomials. Finally, the method is applied. The results are shown in tables and graphs. Accordingly, the approximate solution obtained by the present method is very close to the exact solution. By examining the results, it can be said that the method is successful. Also, results are obtained with the codes written in the Matlab program.

Keywords: Collocation Method, Collocation Points, Fredholm Integro-Differential Equations, Pell-Lucas Polynomials

1. INTRODUCTION

Lately, it is very common in science and engineering to model problems through differential equations and systems of differential equations (Biazar, Tango, Babolian, & Islam, 2003), (Piqueira & Araujo, 2009), (Lutambi, Penny , Smith, & Chitnis, 2013), (Momoniat & Harley, 2011), (Sprott, 2005), (Ghadikolaei, Yassari, Sadeghi , Hosseinzadeh, & Ganji, 2017), (Dogonchi, Hatami, Hosseinzadeh, & Domairry, 2015), (Ghadikolaei, Hosseinzadeh, Yassari, Sadeghi, & Ganji, 2017), (Atouei, ve diğerleri, 2015), (Hatami, Hosseinzadeh, Domairry, & Behnamfar, 2014). Since it is not always possible to calculate analytical solutions of these equations or systems of equations, many numerical methods have been developed (Ghadikolaei, Hosseinzadeh, Yassari, Sadeghi, & Ganji, 2017), (Atouei, ve diğerleri, 2015), (Hatami, Hosseinzadeh, Domairry, & Behnamfar, 2014), (Akyüz & Sezer, 2003), (Gökmen & Sezer, 2013), (Pandey & Kumar, 2012), (Khanian & Davari, 2011), (Öztürk, 2018), (Rabbani & Zarali, 2012), (Akyüz-Daşcıoğlu & Sezer, 2005), (Yalçınbaş, Sezer, & Sorkun, 2009), (Dehghan & Saadatmandi, Chebyshev finite difference method for Fredholm integro-differential equation., 2008), (Kurt & Sezer, 2008), (Maleknejad, Basirat, & Hashemizadeh, 2012), (Mirzaee & Hoseini, 2014), (Yüzbaşı, Şahin, & Sezer, 2011) , (Türkyılmazoğlu, 2014). In addition, there are numerical studies using Pell-Lucas polynomials (Yüzbaşı & Yıldırım, 2020), (Yüzbaşı & Yıldırım, 2020). In this paper, we will study the system of m-order linear Fredholm integral equation

$$\sum_{n=0}^{m}\sum_{j=1}^{k} P_{i,j}^{n}(x)\, y_j^{(n)}(x) = g_i(x) + \int_a^b \sum_{j=1}^{k} K_{i,j}(x,t) y_j(t)\, dt, i = 1,2,\dots,k, 0 \le a \le x \le b \tag{42}$$

with conditions

$$\sum_{j=0}^{m} (a_{i,j}^{n} y_n^{(j)}(a) + b_{i,j}^{n} y_n^{(j)}(b)) = \lambda_{n,i}, \qquad i = 0,1,\dots,m-1, \;\; n = 1,2,\dots,k \tag{43}$$

In this study, we will develop a method to obtain the approximate solutions of the problem *(1)-(2)* by using matrix representation of the Pell-Lucas polynomials. Here, we will look for approximate solutions as

$$y_n^{j}(x) = \sum_{r=0}^{N} a_r^{j}\, Q_n(x) \tag{44}$$

The definitions of the parameters in the statements *(1)-(3)* are given in the Table 1.

Tablo 5 Some expressions in (1)-(2)-(3)

Parameter	**Definition**
$a_{i,j}^{n}, \lambda_{n,i}$	real or complex constants
$y_j^{(n)}(x)$	n. order derivative
$y_j^{(0)}(x) = y_j(x)$	the approximate solution
$P_{i,j}^{n}(x), g_i(x), K_{i,j}(x,t)$	analytical functions
$K_{i,j}(x,t)$	extensible function to Maclaurin series
$Q_n(x)$	the Pell-Lucas polynomials
a_r^{j}	unknown Pell-Lucas coefficients
N	chosen any positive integer

2. PELL-LUCAS POLYNOMIALS

The Pell-Lucas polynomials $Q_n(x)$ is (Horadam & Mahon Bro, 1985) (Horadam, Swita, & Filipponi, 1994)

$$Q_n(x) = \sum_{k=0}^{\lceil n/2 \rceil} 2^{n-2k} \frac{n}{n-k} \binom{n-k}{k} x^{n-2k}. \tag{45}$$

And the recurrence relationship of Pell-Lucas polynomials $Q_n(x)$ is (Horadam & Mahon Bro, 1985) (Horadam, Swita, & Filipponi, 1994)

$$Q_n(x) = 2xQ_{n-1}(x) + Q_{n-2}(x), n \geq 2$$

where the first two Pell-Lucas polynomials are $Q_0(x) = 2$ and $Q_1(x) = 2x$.

Additionally, the derivative relation of Pell-Lucas polynomials $Q_n(x)$ is

$$Q_n'(x) = 2xQ_{n-1}'(x) + Q_{n-2}'(x) + 2Q_{n-1}(x), n \geq 2. \tag{46}$$

We can be approached to the given function $y_n^j(x)$ in the form

$$y_n^j(x) = \sum_{r=0}^{N} a_r^j Q_n(x) \tag{47}$$

where a_r^j is in form

$$a_r^j = \langle y_n^j(x), Q_n(x) \rangle = \int_0^1 y_n^j(x) Q_n(x)\, dx. \tag{48}$$

3. FUNDAMENTAL MATRIX RELATIONSHIPS

We can write the matrix representation of the Equation *(6)* as

$$y_N^j(x) = \mathbf{Q}(x)\mathbf{A}_j. \tag{49}$$

Here, $\mathbf{Q}(x)$ and $\mathbf{A}_j$ are

$$\mathbf{Q}(x) = [Q_0(x) \quad Q_1(x) \quad \cdots \quad Q_N(x)], \mathbf{A}_j = \begin{bmatrix} a_0^j & a_1^j & \cdots & a_N^j \end{bmatrix}^T.$$

The Pell-Lucas polynomials $Q_n(x)$ can be expressed

$$\mathbf{Q}(x) = \boldsymbol{X}(x)\mathbf{D}^{T} \tag{50}$$

where $\mathbf{X}(x) = [1 \quad x \quad \cdots \quad x^{N}]$ and if N is odd

$$\mathbf{D} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 2^{1}\frac{1}{1}\binom{1}{0} & 0 & 0 & 0 & 0 & \cdots & 0 \\ 2^{0}\frac{2}{1}\binom{1}{1} & 0 & 2^{2}\frac{2}{2}\binom{2}{0} & 0 & 0 & 0 & \cdots & 0 \\ 0 & 2^{1}\frac{3}{2}\binom{1}{1} & 0 & 2^{3}\frac{3}{3}\binom{3}{0} & 0 & 0 & \cdots & 0 \\ 2^{0}\frac{4}{2}\binom{2}{2} & 0 & 2^{2}\frac{4}{3}\binom{3}{1} & 0 & 2^{4}\frac{4}{4}\binom{4}{0} & 0 & \cdots & 0 \\ 0 & 2^{1}\frac{5}{3}\binom{3}{2} & 0 & 2^{3}\frac{5}{4}\binom{4}{1} & 0 & 2^{5}\frac{5}{5}\binom{5}{0} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 2^{1}\frac{N}{\frac{N+1}{2}}\binom{\frac{N+1}{2}}{\frac{N-1}{2}} & 0 & 2^{3}\frac{N}{\frac{N+3}{2}}\binom{\frac{N+3}{2}}{\frac{N-3}{2}} & 0 & 2^{5}\frac{N}{\frac{N+5}{2}}\binom{\frac{N+5}{2}}{\frac{N-5}{2}} & \cdots & 2^{N}\frac{N}{N}\binom{N}{0} \end{bmatrix}$$

and if N is even

$$\mathbf{D} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 2^{1}\frac{1}{1}\binom{1}{0} & 0 & 0 & 0 & 0 & \cdots & 0 \\ 2^{0}\frac{2}{1}\binom{1}{1} & 0 & 2^{2}\frac{2}{2}\binom{2}{0} & 0 & 0 & 0 & \cdots & 0 \\ 0 & 2^{1}\frac{3}{2}\binom{1}{1} & 0 & 2^{3}\frac{3}{3}\binom{3}{0} & 0 & 0 & \cdots & 0 \\ 2^{0}\frac{4}{2}\binom{2}{2} & 0 & 2^{2}\frac{4}{3}\binom{3}{1} & 0 & 2^{4}\frac{4}{4}\binom{4}{0} & 0 & \cdots & 0 \\ 0 & 2^{1}\frac{5}{3}\binom{3}{2} & 0 & 2^{3}\frac{5}{4}\binom{4}{1} & 0 & 2^{5}\frac{5}{5}\binom{5}{0} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2^{0}\frac{N}{\frac{N}{2}}\binom{\frac{N}{2}}{\frac{N}{2}} & 0 & 2^{2}\frac{N}{\frac{N+2}{2}}\binom{\frac{N+2}{2}}{\frac{N-2}{2}} & 0 & 2^{4}\frac{N}{\frac{N+2}{2}}\binom{\frac{N+4}{2}}{\frac{N-4}{2}} & 0 & \cdots & 2^{N}\frac{N}{N}\binom{N}{0} \end{bmatrix}$$

Thus, the matrix representation of the relationship between the Equation *(8)* and the derivatives of the Equation *(8)* can be expressed as

$$(y_N^j(x))^{(n)} = \mathbf{Q}^{(n)}(x)\mathbf{A}_j \tag{51}$$

Also by using the expression *(9)*, the matrix representation of the expression $(\mathbf{Q})^{(n)}(x)$ is

$$\mathbf{Q}^{(n)}(x) = \mathbf{X}^{(n)}(x)\mathbf{D}^T \tag{52}$$

and here the $n-th$ derivative of the expression $\mathbf{X}(x)$ is

$$\mathbf{X}^{(n)}(x) = \mathbf{X}(x)(\mathbf{B}^T)^n \tag{53}$$

where $\mathbf{B}$ is denoted as

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \mathrm{N} & 0 \end{bmatrix}.$$

When the expression *(12)* is written in the expression *(11)*, it becomes

$$\mathbf{Q}^{(n)}(x) = \mathbf{X}(x)(\mathbf{B}^T)^n\mathbf{D}^T \tag{54}$$

And when the expression *(13)* is written in the expression *(10)*, it becomes

$$(y_N^j(x))^{(n)} = \mathbf{X}(x)(\mathbf{B}^T)^n\mathbf{D}^T\mathbf{A}_j. \tag{55}$$

Then, for the Fredholm part in the system *(1)*, we can write the kernel function $K_{i,j}(x,t)$ as

$$K_{i,j}(x,t) = \sum_{m=0}^{N}\sum_{n=0}^{N} {}^{t}k_{i,j}^{m,n}\, x^m t^n \tag{56}$$

and

$$K_{i,j}(x,t) = \sum_{m=0}^{N}\sum_{n=0}^{N} {}^{q}k_{i,j}^{m,n}\, Q_m(x)\, Q_n(t) \tag{57}$$

where

$${}^{t}k_{i,j}^{m,n} = \frac{1}{m!\,n!}\frac{\partial^{m+n}K(0,0)}{\partial x^m \partial t^n};\ m,n = 0,1,\ldots,N;\ i = 1,2,\ldots,k.$$

Thus, we can write the expression *(15)* and *(16)* in matrix form as

$$K_{i,j}(x,t) = \mathbf{X}(x)\mathbf{K}_{i,j}^{t}\mathbf{X}^T(t);\ \mathbf{K}_{i,j}^{t} = \left[{}^{t}k_{i,j}^{m,n}\right] \tag{58}$$

and

$$K_{i,j}(x,t) = \mathbf{Q}(x)\mathbf{K}_{i,j}^{q}\mathbf{Q}^{T}(t);\ \mathbf{K}_{i,j}^{q} = \left[{}^{q}k_{i,j}^{m,n}\right]. \tag{59}$$

From the expressions *(17)* and *(18)*, we gain the relation

$$\mathbf{K}_{i,j}^{t} = \mathbf{D}^{T}\mathbf{K}_{i,j}^{q}\mathbf{D} \ \text{ or } \ \mathbf{K}_{i,j}^{q} = (\mathbf{D}^{T})^{-1}\,\mathbf{K}_{i,j}^{t}\mathbf{D}^{-1}. \tag{60}$$

If, we substitute the expressions *(8)* and *(19)* into the expressions *(1)* for the Fredholm part, then it becomes

$$I(x) = \sum_{i=1}^{k}\sum_{j=1}^{k}\int_{a}^{b}\mathbf{Q}(x)\mathbf{K}_{i,j}^{q}\mathbf{Q}^{T}(t)\boldsymbol{X}(t)\mathbf{D}^{T}\,\mathbf{A}_{j}\,dt \tag{61}$$

or

$$I(x) = \sum_{i=1}^{k}\sum_{j=1}^{k}\boldsymbol{X}(x)\mathbf{D}^{T}\mathbf{K}_{i,j}^{q}\mathbf{D}\left\{\int_{a}^{b}\boldsymbol{X}^{T}(t)\boldsymbol{X}(t)dt\right\}\mathbf{D}^{T}\mathbf{A}_{j}. \tag{62}$$

Now, we substitute the expressions *(19)* into the expressions *(21)* and we gain

$$I(x) = \sum_{i=1}^{k}\sum_{j=1}^{k}\boldsymbol{X}(x)\mathbf{K}_{i,j}^{t}\mathbf{N}\mathbf{D}^{T}\mathbf{A}_{j}. \tag{63}$$

where

$$\mathbf{N} = \left\{\int_{a}^{b}\boldsymbol{X}^{T}(t)\boldsymbol{X}(t)dt\right\} = [\boldsymbol{n}_{rs}];\ \boldsymbol{n}_{rs} = \frac{\boldsymbol{b}^{r+s+1} - \boldsymbol{a}^{r+s+1}}{\boldsymbol{r}+\boldsymbol{s}+\mathbf{1}};\ \boldsymbol{r},\boldsymbol{s} = \mathbf{0},\mathbf{1},\ldots,\boldsymbol{N}.$$

Finally, substitute the expression *(14)* and the expression *(22)* in the expression *(1)* for $i = 1,2,\ldots,k$, it is obtained

$$\sum_{n=0}^{m}\sum_{j=1}^{k}P_{i,j}^{n}(x)\,\mathbf{X}(x)(\mathbf{B}^{T})^{n}\mathbf{D}^{T}\mathbf{A}_{j} = g_{i}(x) + \sum_{j=1}^{k}\boldsymbol{X}(x)\mathbf{K}_{i,j}^{t}\mathbf{N}\mathbf{D}^{T}\mathbf{A}_{j}. \tag{64}$$

4. THE OPERATIONAL MATRIX METHOD

In this section, we will be developed a method to obtain approximate solutions of the system of the Equation *(1)* under the conditions *(2)*. So, we give the expression $g_i(x)$ in the form

$$g_i(x) \approx \mathbf{X}(x)\mathbf{D}^{T}\mathbf{G}_{i}^{T} \tag{65}$$

depending on the Pell-Lucas polynomials. Then, we write the expression *(24)* in the expression *(23)* for $i = 1,2,\ldots,k$, as

$$\sum_{n=0}^{m}\sum_{j=1}^{k} P_{i,j}^{n}(x)\,\mathbf{X}(x)(\mathbf{B}^T)^n\mathbf{D}^T\mathbf{A}_j - \sum_{j=1}^{k} \boldsymbol{X}(x)\mathbf{K}_{i,j}^{t}\mathbf{N}\mathbf{D}^T\mathbf{A}_j \approx \mathbf{X}(x)\mathbf{D}^T\mathbf{G}_i^T. \tag{66}$$

Now, we write the residual $R_i(x)$ for $i = 1,2,\ldots,k$, as

$$R_i(x) \approx \sum_{n=0}^{m}\sum_{j=1}^{k} P_{i,j}^{n}(x)\,\mathbf{X}(x)(\mathbf{B}^T)^n\mathbf{D}^T\mathbf{A}_j - \sum_{j=1}^{k} \boldsymbol{X}(x)\mathbf{K}_{i,j}^{t}\mathbf{N}\mathbf{D}^T\mathbf{A}_j - \mathbf{X}(x)\mathbf{D}^T\mathbf{G}_i^T. \tag{67}$$

Consequently, we use the Tau method and we convert the expression *(27)* to the *m(N-1)* linear equations by applying

$$\langle R_i(x), Q_n(x)\rangle = \int_0^1 R_i(x)Q_n(x)\;dx = 0,\;\; n = 0,1,\ldots,N-2. \tag{68}$$

Similarly, we substitute the expressions *(14)* into the expressions *(2)* for $n = 1,2,\ldots,k$ and $i = 0,1,\ldots,m-1$ and we gain

$$\sum_{j=0}^{m-1}\left(a_{i,j}^{n}\mathbf{X}(a)(\mathbf{B}^T)^j\mathbf{D}^T\mathbf{A}_j + b_{i,j}^{n}\mathbf{X}(b)(\mathbf{B}^T)^j\mathbf{D}^T\mathbf{A}_j\right) = \lambda_{n,i}. \tag{69}$$

The Equations *(27)* and *(28)* generate $m(N+1)$ sets of linear equations. By solving this system with the help of Matlab, the coefficient matrix $\mathbf{A}_j$ and the solutions $y_N^j(x)$ are obtained.

5. NUMERICAL EXAMPLE

In this section, we will apply the methods in Section 3 and Section 4. We have made the calculations for these applications in Matlab program. Thus, we will see that the method is effective and reliable.

Example 5.1. We consider the system

$$\begin{cases} y_1''(x) + x\,y_1(x) + xy_2(x) = 2 + \displaystyle\int_{-1}^{1}(y_1(t)+y_2(t))\,dt \\ y_2''(x) + 2x\,y_2(x) + 2xy_1(x) = -2 + \displaystyle\int_{-1}^{1}(2y_1(t)+2y_2(t))\,dt \end{cases} \tag{70}$$

and the conditions

$$y_1(0) = y_1(1) = 0, \ \ y_2(0) = y_2(1) = 0. \tag{71}$$

Since we applied the method in Section 3-4 for $N = 2$, we can write

$$\begin{gathered}(y_2^1)''(x) = a_0^1 Q_0(x) + a_1^1 Q_1(x) + a_2^1 Q_2(x)\\ (y_2^2)''(x) = a_0^2 Q_0(x) + a_1^2 Q_1(x) + a_2^2 Q_2(x)\end{gathered} \tag{72}$$

and the residual of this problem becomes

$$\begin{aligned}\mathbf{R}_1(x) \approx \ & \mathbf{X}(x)(\mathbf{B}^T)^2\mathbf{D}^T\mathbf{A}_1 + x\mathbf{X}(x)\mathbf{D}^T\mathbf{A}_1 + x\mathbf{X}(x)\mathbf{D}^T\mathbf{A}_2 - \boldsymbol{X}(x)\mathbf{K}_{\mathbf{1,1}}^{\boldsymbol{t}}\mathbf{N}\mathbf{D}^T\mathbf{A}_1\\ & - \boldsymbol{X}(x)\mathbf{K}_{\mathbf{1,2}}^{\boldsymbol{t}}\mathbf{N}\mathbf{D}^T\mathbf{A}_2 - \mathbf{X}(x)\mathbf{D}^T\mathbf{G}_1^T\end{aligned} \tag{73}$$

and

$$\begin{aligned}\mathbf{R}_2(x) \approx \ & \mathbf{X}(x)(\mathbf{B}^T)^2\mathbf{D}^T\mathbf{A}_2 + 2x\mathbf{X}(x)\mathbf{D}^T\mathbf{A}_2 + 2x\mathbf{X}(x)\mathbf{D}^T\mathbf{A}_1 - \boldsymbol{X}(x)\mathbf{K}_{\mathbf{2,1}}^{\boldsymbol{t}}\mathbf{N}\mathbf{D}^T\mathbf{A}_1\\ & - \boldsymbol{X}(x)\mathbf{K}_{\mathbf{2,2}}^{\boldsymbol{t}}\mathbf{N}\mathbf{D}^T\mathbf{A}_2 - \mathbf{X}(x)\mathbf{D}^T\mathbf{G}_2^T.\end{aligned} \tag{74}$$

where

$$\mathbf{D} = \begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 2 & 0 & 4\end{bmatrix}, \mathbf{B} = \begin{bmatrix}0 & 0 & 0\\ 1 & 0 & 0\\ 0 & 2 & 0\end{bmatrix}, \mathbf{X}(x) = \begin{bmatrix}1\\ x\\ x^2\end{bmatrix}^T, \mathbf{G}_{\mathbf{1}}^{\boldsymbol{T}} = \begin{bmatrix}2\\ 0\\ 0\end{bmatrix}, \mathbf{G}_{\mathbf{2}}^{\boldsymbol{T}} = \begin{bmatrix}-2\\ 0\\ 0\end{bmatrix},$$

$$\mathbf{A}_1 = \begin{bmatrix}a_0^1\\ a_1^1\\ a_2^1\end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix}a_0^2\\ a_1^2\\ a_2^2\end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix}2 & 0 & 2/3\\ 0 & 2/3 & 0\\ 2/3 & 0 & 2/5\end{bmatrix}, \quad \mathbf{K}_{11}(x) = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$$\mathbf{K}_{12}(x) = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, \mathbf{K}_{21}(x) = \begin{bmatrix}2 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, \mathbf{K}_{22}(x) = \begin{bmatrix}21 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}.$$

Thus, we use the Equations *(32)* and *(33)* and we gain

$$-6a_0^1 + \frac{4}{3}a_1^1 + \frac{20}{3}a_2^1 - 6a_0^2 + \frac{4}{3}a_1^2 - \frac{28}{3}a_2^2 = 4 \tag{75}$$

$$-12a_0^1 + \frac{8}{3}a_1^1 - \frac{56}{3}a_2^1 - 12a_0^2 + \frac{8}{3}a_1^2 - \frac{8}{3}a_2^2 = -4 \tag{76}$$

and for conditions we gain

$$\begin{gathered}y_2^1(0) = \mathbf{X}(0)\mathbf{D}^T\mathbf{A}_1 = 2a_0^1 + 2a_2^1 = 0\\ y_2^1(1) = \mathbf{X}(1)\mathbf{D}^T\mathbf{A}_1 = 2a_0^1 + 2a_1^1 + 6a_2^1 = 0\\ y_2^2(0) = \mathbf{X}(0)\mathbf{D}^T\mathbf{A}_2 = 2a_0^2 + 2a_2^2 = 0\\ y_2^2(1) = \mathbf{X}(1)\mathbf{D}^T\mathbf{A}_2 = 2a_0^2 + 2a_1^2 + 6a_2^2 = 0\end{gathered} \tag{77}$$

Hence, we solve the system *(34)*,*(35)*,*(36)* and we obtain

$$a_0^1 = -\frac{1}{4}, a_1^1 = -\frac{1}{2}, a_2^1 = \frac{1}{4}, a_0^2 = \frac{1}{4}, a_1^2 = \frac{1}{2}, a_2^2 = -\frac{1}{4}. \quad (78)$$

We put these coefficients in the Equation *(31)*, and so we gain the approximate solutions as

$$y_2^1(x) = -5.551115123125783e - 17 - 9.999999962747099e - 01\,x + 9.999999962747100e - 01\,x^2, \quad (79)$$

$$y_2^2(x) = 1.000000011175870e + 00x - 1.000000011175870e + 00x^2. \quad (80)$$

Here, the approximate solution and the absolute errors for $y_2^1(x)$ are given in Table 2 and Figure 1. And the approximate solution and the absolute errors for $y_2^2(x)$ are given in Table 3 and Figure 2. Also, the absolute errors for $N = 2, 5$ are given in Table 3. When all these tables and graphs are examined, it can be said that the method is effective.

Tablo 6 The exact solution, the approximate solution and the actual absolute error of the system *(29)-(30)* for $y_2^1(x)$

	Exact solution	**Approximate Solution**	**Actual Absolute Error**
$\boldsymbol{x_i}$	$y(x_i) = -x + x^2$	$N = 2, \quad y_2^1(x_i)$	$N = 2, \quad e_2^1(x_i)$
$\mathbf{0}$	0	$-5.551115123125783e - 17$	$5.5511e - 17$
$\mathbf{0.2}$	-0.16	-0.159999999403954	$5.9605e - 10$
$\mathbf{0.4}$	-0.24	-0.239999999105930	$8.9407e - 10$
$\mathbf{0.6}$	-0.24	-0.239999999105930	$8.9407e - 10$
$\mathbf{0.8}$	-0.16	-0.159999999403954	$5.9605e - 10$
$\mathbf{1}$	0	$5.551115123125783e - 17$	$5.5511e - 17$

Tablo 7 The exact solution, the approximate solution and the actual absolute error of the system (29)-(30) for $y_2^2(x)$

	Exact solution	**Approximate Solution**	**Actual Absolute Error**
$\boldsymbol{x_i}$	$y(x_i) = x - x^2$	$N = 2, \quad y_2^2(x_i)$	$N = 2, \quad e_2^2(x_i)$

0	0	0	0
0.2	0.16	0.160000001788139	$1.7881e-09$
0.4	0.24	0.240000002682209	$2.6822e-09$
0.6	0.24	0.240000002682209	$2.6822e-09$
0.8	0.16	0.160000001788139	$1.7881e-09$
1	0	0	0

Tablo 8 The actual absolute error of the system (29)-(30)

	Actual Absolute Error			
$\boldsymbol{x_i}$	$N=2$, $e_2^1(x_i)$	$N=2$, $e_2^2(x_i)$	$N=5$, $e_5^1(x_i)$	$N=5$, $e_5^2(x_i)$
0	$5.5511e-17$	0	1.5828e-17	1.1744e-17
0.2	$5.9605e-10$	$1.7881e-09$	8.5573e-14	1.6507e-13
0.4	$8.9407e-10$	$2.6822e-09$	1.3161e-13	2.5128e-13
0.6	$8.9407e-10$	$2.6822e-09$	1.3258e-13	2.5503e-13
0.8	$5.9605e-10$	$1.7881e-09$	8.7871e-14	1.7162e-13
1	$5.5511e-17$	0	2.1519e-16	6.3068e-17

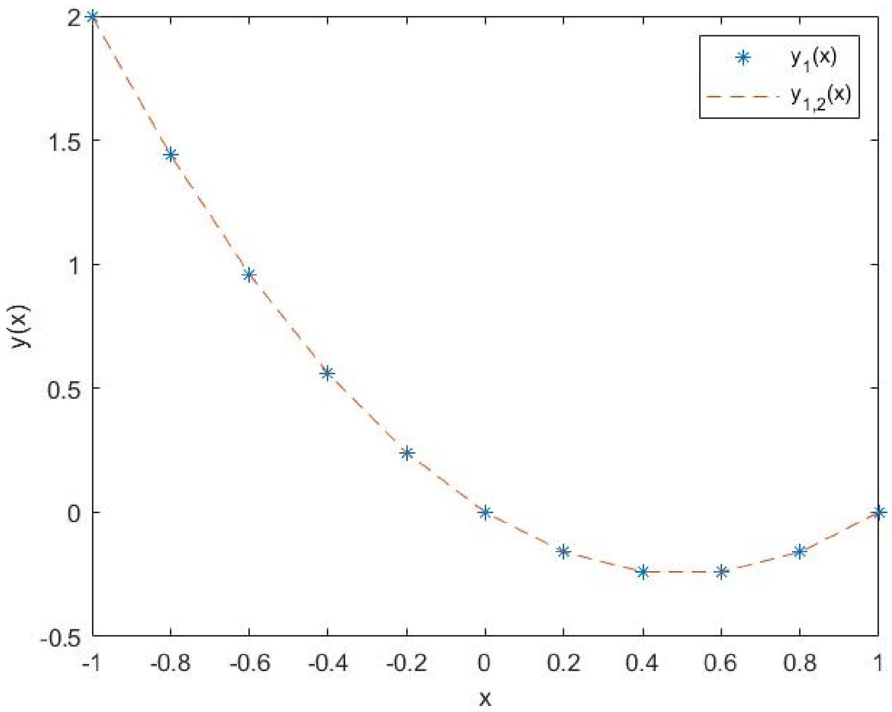

Figure 3 The exact solution and the approximate of the system *(29)-(30)* for $y_2^1(x)$

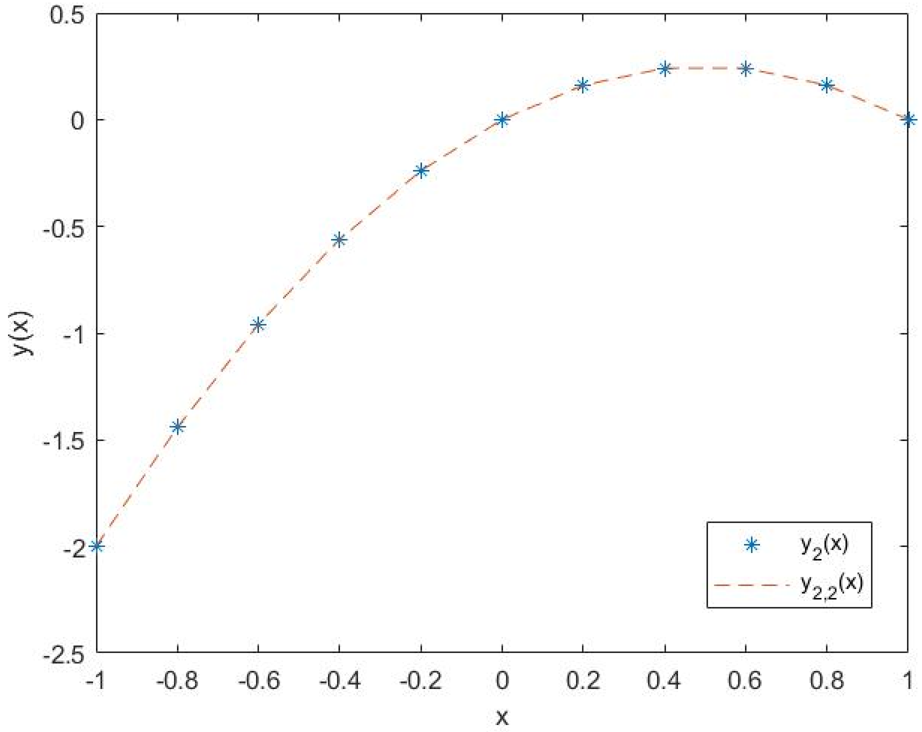

Figure 4 The exact solution and the approximate of the system (30)-(31) for $y_2^2(x)$

6. CONCLUSIONS

In this work, we developed a method based on Pell-Lucas polynomials that solves numerically the system of linear Fredholm integral equation. For the method, the problem was reduced to the algebraic equation system. The coefficients of this system were determined depending on the Pell-Lucas polynomials. The solution of this system give us the coefficients of the approximate solution. Thus, approximate solutions were obtained based on Pell-Lucas polynomials. The application of the method was also made and it was seen with the help of tables and graphics that the method was effective. The method can also be improved for the Volterra integral part if the necessary adjustments are made.

REFERENCES

Akyüz, A., & Sezer, M. (2003). Chebyshev polynomial solutions of systems of high order linear differential equations with variable coefficients. *Appl Math Comput, 144*, 237-247.

Akyüz-Daşcıoğlu, A., & Sezer, M. (2005). Chebyshev polynomial solutions of systems of higher-order linear Fredholm-Volterra integro-differential equations. *J. Franklin Inst., 342*, 688-701.

Atouei, S., Hosseinzadeh, K., Hatami, M., Ghasemi, S., Sahebi, S., & Ganji, D. (2015). Heat transfer study on convective–radiative semi-spherical fins with temperature-dependent properties and heat generation using efficient computational methods. *Appl Therm Eng., 89*, 299-305.

Biazar, J., Tango, M., Babolian, E., & Islam, R. (2003). Solution of the kinetic modeling of lactic acid fermentation using Adomian decomposition method. *Appl Math Comput., 139*, 249-258.

Dehghan, M., & Saadatmandi, A. (2008). Chebyshev finite difference method for Fredholm integro-differential equation. *International Journal of Computer Mathematics, 85*, 123-130.

Dogonchi, A., Hatami, M., Hosseinzadeh, K., & Domairry, G. (2015). Non-spherical particles sedimentation in an incompressible Newtonian medium by Padé approximation. *Powder Technol., 278*, 248-256.

Ghadikolaei, S., Hosseinzadeh, K., Yassari, M., Sadeghi, H., & Ganji, D. (2017). Boundary layer analysis of micropolar dusty fluid with TiO_2 nanoparticles in a porous medium under the effect of magnetic field and thermal radiation over a stretching sheet. *J Mol Liq., 244*, 374-389.

Ghadikolaei, S., Yassari, M., Sadeghi , H., Hosseinzadeh, K., & Ganji, D. (2017). Investigation on thermophysical properties of TiO_2-Cu/H_2O hybrid nanofluid transport dependent on shape factor in MHD stagnation point flow. *Powder Technol., 322*, 428-438.

Gökmen, E., & Sezer, M. (2013). Taylor collocation method for systems of high-order linear differential–difference equations with variable coefficients. *Ain Shams Eng J., 4*(1), 117-125.

Hatami, M., Hosseinzadeh, K., Domairry, G., & Behnamfar, M. (2014). Numerical study of MHD two-phase Couette flow analysis for fluid-particle suspension between moving parallel plates. *J Taiwan Inst Chem Eng., 45,* 2238-2245.

Horadam, A., & Mahon Bro, J. (1985). Pell and Pell-Lucas Polynomials. *The Fibonacci Quarterly, 23*, 7-20.

Horadam, A., Swita, B., & Filipponi, P. (1994). Integration and Derivative Sequences for Pell and Pell-Lucas Polynomials. *The Fibonacci Quarterly, 32*, 130-35.

Khanian, M., & Davari, A. (2011). Solution of System of Fredholm Integro-Differential Equations By Adomian Decomposition Method. *Australian Journal of Basic and Applied Sciences*, *5*(12), 2356-2361.

Kurt, N., & Sezer, M. (2008). Polynomial solution of high-order linear Fredholm integro-differential equations with constant coefficients. *Journal of the Franklin Institute, 345*, 839-850.

Lutambi, A., Penny , M., Smith, T., & Chitnis, N. (2013). Mathematical modelling of mosquito dispersal in a heterogeneous environment. *Math Biosci., 241*, 198-216.

Maleknejad, K., Basirat, B., & Hashemizadeh, E. (2012). A Bernstein operational matrix approach for solving a system of high order linear Volterra-Fredholm integro-differential equations. *Mathematical and Computer Modelling, 55*, 1363-1372.

Mirzaee, F., & Hoseini, S. (2014). Solving systems of linear Fredholm integro-differential equations with Fibonacci polynomials. *Ain Shams Engineering Journal*, *5*, 271-283.

Momoniat, E., & Harley, C. (2011). An implicit series solution for a boundary value problem modelling a thermal explosion. *Math Comput Model.*, *53*, 249-260.

Öztürk, Y. (2018). Numerical Solution of Systems of Differential Equations Using Operational Matrix Method with Chebyshev Polynomials. *Journal of Taibah University for Science, 12*(2), 155-162.

Pandey, R., & Kumar, N. (2012). Solution of lane-Emden type equations using Berstein operational matrix of differentiation. *New Astron., 17*, 303-308.

Piqueira, J., & Araujo, V. (2009). A modified epidemiological model for computer viruses. *Appl Math Comput., 213*, 355-360.

Rabbani, M., & Zarali, B. (2012). Solution of Fredholm Integro-Differential Equations System by Modified Decomposition Method. *The Journal of Mathematics and Computer Science, 5,* 258-264.

Sprott, J. (2005). Dynamical models of happiness. *Nonlinear Dyn Psychol Life Sci.*, *9*(1), 23-36.

Türkyılmazoğlu, M. (2014). An effective approach for numerical solutions of high-order Fredholm integro-differential equations. *Applied Mathematics and Computation, 227*, 384-398.

Yalçınbaş, S., Sezer, M., & Sorkun, H. (2009). Legendre polynomial solutions of high-order linear Fredholm integro-differential equations. *Applied Mathematics and Computation, 210*, 334-349.

Yüzbaşı, Ş., & Yıldırım, G. (2020). Pell–Lucas collocation method for numerical solutions of two population models and residual correction. *Journal of Taibah University for Science, 14:1*, 1262-1278.

Yüzbaşı, Ş., & Yıldırım, G. (2020). Pell-Lucas collocation method to solve high-order linear Fredholm-Volterra integro-differential equations and residual correction. *Turkish Journal of Mathematics, 44*, 1065-1091.

Yüzbaşı, Ş., Şahin, N., & Sezer, M. (2011). Numerical solutions of systems of linear Fredholm integro-differential equations with Bessel polynomial bases. *Computers & Mathematics with Applications, 61*, 3079-3096.

MATHEMATICS EDUCATION CREATING FEAR AND MISCONCEPTION

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PONDA-GOA

Mathematics is the most misunderstood, hated and feared subject. The need of the subject and its role in human life is scarcely clear to people, in general. The damage is done in the teaching of the subject by emphasis on the manipulation of symbols and getting the correct answer swiftly. Mathematics learning needs to embrace the meaning of the subject rather than play with symbols for marks. The understanding of the problem and the concepts have been relegated and replaced with extreme emphasis to speed of finding the 'right answer'. The very crux of mathematics teaching is to develop problem solving skills and to apply it in real life context. It is supposed to make humans think and rationalize.

Cobb et al. (1991) suggested, the purpose for engaging in problem solving is not just to solve specific problems, but to 'encourage the interiorization and reorganization of the involved schemes as a result of the activity'. Schoenfeld(1994) opines the conventional learning of mathematics only enables students to perform algorithmically and understand mathematics without reasoning, Jenning and Dunne (1999)have expressed the view that most students have difficulty in applying mathematics in real-world situations and Van den Heuvel-Panhuizen (1988) argues that students will most likely fail to remember the concepts and will be unable to apply mathematical concepts.

In this paper the discussion will be about the wrong understanding of mathematics developed by faulty teaching methods. The amount of fear and misconception due to formal education and emphasis on knowing the algorithm while street children and semi-literates use mathematics with proficiency.

REFERENCES

Cobb, P., Wood, T. and Yackel, E. (1991). 'A constructivist approach to second grade mathematics'. In von Glaserfield, E. (Ed.), Radical Constructivism in Mathematics Education, pp. 157-176. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Jennings, S. and Dunne, R.(1999), Match Stories Real Stories, Real-life Stories. http://www.ex.ac.uk/telematics/T3/maths/actar01.htm.

Panhuizen, V.D.H.(1988). Realistic Mathematics Education Work in Progress. Mathematics Education in the Netherlands: a Guided Tour. http://edweb.gsn.org/edref.sys.learn.html.

Schoenfeld, A.H.(1992).On Mathematics as Sense-Making: An Informal Attach on Unfortunate Deverce of Formal and Informal Mathematics. In: Coss, J.F., Perkins, D.N. and Segal, J.W. (Eds), Informal Reasoning and Education. Hillsdale, NJ: Erlabum, 311-344.

STATISTICAL KOROVKIN AND VORONOVSKAYA TYPE THEOREM FOR THE CESARO SECOND-ORDER OPERATOR OF FUZZY NUMBERS

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ABSTRACT

In this paper we de_ne the Ces_aro second-order summability method for fuzzy numbers and prove Korovkin type theorem, then as the application of it, we prove the rate of convergence. In the last section, we prove the kind of Voronovskaya type theorem and give some concluding remarks related to the obtained results. Mathematics Subject Classi_cation (2010): 40A10, 40C10, 40E05, 40A05, 40G99, 26E50.

Keywords: Ces_aro second order summability method, statistical convergence,

Korovkin type theorem, rate of convergence, Voronovskaya type theorem.

REFERENCES

[1] Altin, Y., Mursaleen, M., Altinok, H., Statistical summability (C; 1) for sequences of fuzzy real numbers and a Tauberian theorem, Journal of Intelligent and Fuzzy Systems, 21(2010), 379-384.

[2] Anastassiou, G.A., Rate of convergence of fuzzy neural network operators, univariate case, J. Fuzzy Math., 10(2002), no. 3, 755-780.

[3] Anastassiou, G.A., On basic fuzzy Korovkin theory, Stud. Univ. Babe_s-Bolyai Math., 50(2005), 3-10.

[4] Anastassiou, G.A., Duman, O., Statistical fuzzy approximation by fuzzy positive linear operators, Computers and Mathematics with Applications, 55(2008), 573-580.

[5] Bede, B., Gal, S.G., Almost periodic fuzzy number valued functions, Fuzzy Sets and Systems, 147(2004), 385-403.

[6] Braha, N.L., Geometric properties of the second-order Ces_aro spaces, Banach J. Math. Anal., 10(2016), no. 1, 1-14.

[7] Braha, N.L., Some weighted equi-statistical convergence and Korovkin type-theorem, Results Math., 70(2016), no. 3-4, 433-446.

[8] Braha, N.L., Loku, V., Srivastava, H.M., _2-weighted statistical convergence and Korovkin and Voronovskaya type theorems, Appl. Math. Comput., 266(2015), 675-686.

[9] Braha, N.L., Srivastava, H.M., Mohiuddine, S.A., A Korovkin's type approximation theorem for periodic functions via the statistical summability of the generalized de la Vallee Poussin mean, Appl. Math. Comput., 228(2014), 162-169.

[10] Fast, H., Sur la convergence statistique, Colloq. Math., 2(1951), 241-244.

[11] Kadak, U., Braha, N.L., Srivastava, H.M., Statistical weighted B-summability and its applications to approximation theorems, Appl. Math. Comput., 302(2017), 80-96.

[12] Loku, V., Braha, N.L., Some weighted statistical convergence and Korovkin type-theorem, J. Inequal. Spec. Funct., 8(2017), no. 3, 139-150.

[13] Matloka, M., Sequence of fuzzy numbers, BUSEFAL, 28(1986), 28-37.

[14] Nanda, S., On sequence of fuzzy numbers, Fuzzy Sets and Systems, 33(1989), 123-126.

[15] Puri, M.L., Ralescu, D.A., Di_erentials of fuzzy functions, J. Math. Anal. Appl., 91 (1983), no. 2, 552-558.

[16] Zadeh, L.A., Fuzzy sets, Information and Control, 8(1965), 338-353.

YÜKSEK MERTEBEDEN HOMOJEN OLMAYAN BİR ADİ DİFERANSİYEL DENKLEMİN NÜMERİK YÖNTEMLE ÇÖZÜMÜ ÜZERİNE

ON SOLUTIONS OF A HIGHER ORDER NONHOMOGENEOUS ORDINARY DIFFERENTIAL EQUATION WITH a NUMERICAL METHOD

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ÖZET

Yüksek mertebeden diferansiyel denklemler modelleme sürecinde önemli bir role sahiptir. Çözüm için hangi yöntemin kullanıldığı da bir o kadar önemlidir. Bu çalışmada, homojen olmayan bir başlangıç değeri probleminin yaklaşık çözümünü elde etmek için, üretici çekirdekli Hilbert uzayı metodu kullanılmıştır.. Üretici çekirdek fonksiyonu elde edilmiş, üzerinde çalışılan problem homojen hale getirilmiştir. Sonuçlar grafiklerle sunulup, mutlak hatalar ve göreceli hatalar tablolar halinde verilmiştir.

Anahtar Kelimeler: Üretici Çekirdek Yöntemi, Homojen Olmayan Adi Diferansiyel Denklemler, Başlangıç Değer Problemi, Yaklaşık Çözüm

ABSTRACT

Higher order differential equations (ODE) has a vital role in the modelling. It is also important which method is used for the solution. In this study, with the purpose of obtaining the approximate solution of a nonhomogeneous initial value problem, reproducing kernel Hilbert space method is used. Reproducing kernel function has been obtained and the problem transformed to the homogeneous form by using a special transformation function. The results have been presented with the graphics. Absolute errors and relative errors have been given in the tables.

Keywords: Reproducing Kernel Method, Nonhomogeneous Ordinary Differential Equations, Initial Value Problems, Approximate Solution

IMPULSIVE FRACTIONAL DIFFERENTIAL EQUATIONS INVOLVING THE HADAMARD FRACTIONAL DERIVATIVE

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ABSTRACT

In this paper, we establish sufficient conditions for the existence of solutions of a class of initial value problems for impulsive fractional differential equations involving the Hadamad fractional derivative of order $0 < r \leq 1$. These results are based on fixed point theorems.

Key words: Initial value problem, fractional differential equation, impulsive, Hadamard fractional derivative, fractional integral, fixed point theorem.

AMS Subject Classification: 26A33, 34A37

1 INTRODUCTION

For $0 < r \leq 1$; this paper deals with the existence of solutions of initial value problems (IVP for short), for the impulsive fractional order differential equation,

${}^{H}_{r}\mathrm{D}y(t) = f(t;\ y(t))$; for almost each $t \in J = [1;\ T]$; (1.1)

$\Delta y \backslash t = t_k = I_k\left(y(t_k^-)\right)$; k = 1,… , m; (1.2)

y(1) = 0; (1.3)

where ${}^{H}_{r}\mathrm{D}$ is the Hadamard fractional derivative, $f : J \times R \to R$ is a continuous function, I_k: R →R, k = 1,…, m; $1 = t_0 < t_1 < \ldots < t_m < t_{m+1} = T$;_$\Delta y \backslash t = t_k = y(t_k^+) - y(t_k^-)$, $y(t_k^+) = \lim_{h \to 0^+}(y(t_k + h))$, and $y(t_k^-) = \lim_{h \to 0^-}(y(t_k + h))$ represent the right and left limits of y(t) at t = tk, k = 1, … ,m.

2 PRELIMINARIES

2.1 Notations and Definitions

In this section, we introduce notations, definitions, and preliminary facts that are used in the remainder of this paper.

Let J = [1; T] be a compact interval, C([1; T];R) be the

$$\|y\|_\infty = sup\{|y(t)|: 1 \leq t \leq T\},$$

and we denote by L^1 ([1; T];R) the Banach space of functions y : [1; T] → R that are Lebesgue integrable with norm

$$\|y\|_{L^1} = \int_1^T |y(t)|dt.$$

AC([1; T],R) is the space of functions y : [1; T] →R, which are absolutely continuous.

Theorem 2.1 (Arzela-Ascoli theorem) Let A be a subset of C(J;E) ; A is relatively compact in C(J;E) if and only if the following conditions are met:

(a) The set A is bounded ie :

$$\exists k > 0: \|f(x)\| \leq k, \forall\ x \in J \ and\ \forall\ f \in A.$$

(b) Set A is equicontinuous ie :

$$\forall \varepsilon > 0, \exists \delta > 0: |t_1 - t_2| < \delta \Rightarrow \|f(t_1) - f(t_2)\| \leq \varepsilon\ for\ all\ t_1, t_{2,} \in J\ and\ all\ f \in A.$$

(c) For all $x \in J: set\ \{f(x), f \in A\} \subset E$ is relatively compact.

Definition 2.2 The Hadamard fractional integral of order r for a function h : [1;+∞)→ R is defined as

$$I^r h(t) = \frac{1}{\Gamma(r)} \int_1^t (\log \frac{t}{s})^{r-1} \frac{h(s)}{s} ds,\ \ r > 0,$$

provided the integral exists.

Definition 2.3 For a function h given on the interval [1;+∞); the r Hadamard fractional-order derivative of h, is defined by

$$\left({}_r^H D h(t)\right) = \frac{1}{\Gamma(n-r)} \left(t \frac{d}{dt}\right)^n \int_1^t \left(\log \frac{t}{s}\right)^{n-r-1} \frac{h(s)}{s} ds, n-1 < r < n, n = [r] + 1.$$

Here n = [r] + 1 and [r] denotes the integer part of r and log(.) = loge(.).

2.2 Fixed point theorem

Theorem 2.4 [Banach's theorem] Let X a Banach space. N : X →X is a contracting operator, Then there is a single fixed point .

Theorem 2.5 [Schaefer's theorem] Let X be a Banach space and N : X → X completely continuous operator. If the set

$$\mathrm{E(N)} = \{\mathrm{x} \in \mathrm{X}: \mathrm{x} = \lambda \mathrm{Nx}\ \mathrm{for}\ \lambda \in [0,1]\},$$

is bounded, then N has fixed points.

Theorem 2.6 [Nonlinear alternative of Leray Schauder] Let X be a Banach space and C a nonempty convex subset of X. Let U a nonempty open subset of C with 0 ∈U and T : U →C continuous and compact operator.

Then either

(a) T has fixed points. Or

(b) There exist u ∈ ∂U and λ ∈ [0; 1] with x = λT(x).

3 EXISTENCE OF SOLUTIONS

Consider the set of functions

PC(J;R) ={ y : J → R, y ∈ C((tk; tk+1],R); k = 1, … , m; and there exist $y(t_k^+)$ *and* $y(t_k^-), k = 1, \dots, m,$ *with* $y(t_k^+) = y(t_k)\}$.

This set is a Banach space with the norm

$$\|y\|_{PC} = sup\{|y(t)|: 1 \le t \le T\}$$

And let

PC'(J;R) ={ y : J → R, y ∈ *AC*((tk; tk+1],R); k = 1, … , m; and there exist $y(t_k^+)$ *and* $y(t_k^-), k = 1, \dots, m,$ *with* $y(t_k^+) = y(t_k)\}$.

This set is a Banach space with the norm

$$\|y\|_{PC'} = sup\{|y(t)|: 1 \le t \le T\},$$

Set

$$J' = J\backslash\{t_1, \dots, t_m\}.$$

Definition 3.1 A function y ∈PC(J;R) ∩PC'(J;R) is said to be a solution of (1.1)-(1.3), if y satisfies ${}^{H}_{r}Dy(t) = f(t; y(t))$ on J'and satisfies conditions (1.2)-(1.3).

To prove the existence of solutions to (1.1)-(1.3), we need the following auxiliary lemma.

Lemma 3.2 Let $0 < r \le 1$ and let $\rho \in$ C(J';R) ∩AC(J';R). A function y is a solution of the fractional integral equation

y(t) =$\frac{1}{\Gamma(r)}\int_1^t (\log\frac{t}{s})^{r-1}\frac{\rho(s)}{s}ds$; if t ∈ [1; t1]

$(\frac{\log(t-tk)}{logtk})^{\alpha-1}\left[\frac{1}{\Gamma(r)}\sum_{k=1}^{m}\int_{t_{k-1}}^{t_k}\left(\log\frac{t}{s}\right)^{r-1}\frac{\rho(s)}{s}ds + \sum_{k=1}^{m} I_k\big(y(t_k)\big)\right] + \frac{1}{\Gamma(r)}\int_1^t(\log\frac{t}{s})^{r-1}\frac{\rho(s)}{s}ds;$
if t ∈ (tk; tk+1]; k=1,. . . ,m.

if and only if y is a solution of the impulsive fractional IVP

${}^{H}_{r}$Dy(t) = ρ(t); for each t ∈J'; (3.2)

$\Delta y \backslash t = t_k = I_k(y(t_k^-))$; k = 1,… , m; (3.3)

y(1) = 0 ; (3.4)

Our firrst result is based on the Banach fixed point theorem.

Theorem 3.3 Assume the following conditions hold:

(H1) There exists a constant l > 0 sush that

|f(t, u) - f(t, ; $\bar{u}$)| ≤l |u-$\bar{u}$| for each t ∈ J and u; $\bar{u}$ ∈ R:

(H2) There exists a constant l*> 0 such that

|Ik(u) -Ik(; $\bar{u}$)| ≤ l* |u-$\bar{u}$| for each u; $\bar{u}$ ∈ R and k = 1,…, m:

If [$\frac{l(m+1)(logT)^r}{\Gamma(r+1)}$ + ml*] < 1; (3.5)

then (1.1)-(1.3) has a unique solution on J.

Proof: To show the existence and the uniqueness of the solution of the problem (1.1)-(1.3) it suffices to verify the fixed point hypotheses of Banach.

We define the operator F : PC(J;R) →PC(J;R) by

$$F(y)(t)=\left(\frac{\log(t-tk)}{logtk}\right)^{\alpha-1}\left[\frac{1}{\Gamma(r)}\sum_{k=1}^{m}\int_{t_{k-1}}^{t_k}\left(\log\frac{t}{s}\right)^{r-1}\frac{\rho(s)}{s}ds+\sum_{k=1}^{m}I_k(y(t_k))\right]$$

$$+\frac{1}{\Gamma(r)}\int_1^t(\log\frac{t}{s})^{r-1}\frac{\rho(s)}{s}ds$$

The fixed points of the operator F are solutions of the problem (1.1)-(1.3).

Let x; y ∈ PC(J;R) and t ∈ J

$|F(x)(t) - F(y)(t)| \leq [\frac{l(m+1)(logT)^r}{\Gamma(r+1)}$ + ml*] $\|x - y\|_\infty$

We will now prove that F is a strict contraction by Banach's theorem.

2 Our second result is based on Schaefer's fixed point theorem.

Theorem 3.4 Assume the following conditions hold:

(H3) The function f : J ×R →R is continuous.

(H4) There exists a constant M > 0 such that $|f(t,u)| \leq M$ for each t ∈J and each u ∈ R:

(H5) The functions Ik : R → R are continuous and there exists a constant M* > 0 such that

|Ik(u)| ≤M* for each u ∈ R; k = 1,…, m:

Then (1.1)-(1.3) has at least one solution on J.

Proof:

We will use Schaefer's fixed point theorem to prove that F has a fixed point. The proof will be given in several steps.

Step 1 F is continuous.

Let yn be a sequence such that yn →y in PC(J;R). For all t ∈ J,

$$|F(y_n)(t) - F(y)(t)|$$
$$\leq (\frac{\log(t-\mathrm{tk})}{log\mathrm{tk}})^{\alpha-1}\left[\frac{1}{\Gamma(r)}\sum_{k=1}^{m}\int_{t_{k-1}}^{t_k}\left(\log\frac{t_k}{s}\right)^{r-1}\frac{|f(s,y_n(s)) - f(s,y(s)|}{s}ds\right.$$
$$\left.+\sum_{k=1}^{m}|I_k(y_n(t_k^-)) - I_k(y(t_k^-))\,|\right] + \frac{1}{\Gamma(r)}\int_1^t(\log\frac{t}{s})^{r-1}\frac{|f(s,y_n(s)) - f(s,y(s)|}{s}ds$$

Since f and Ik; k = 1,…, m; are continuous functions, we have

$$\|F(y_n) - F(y)\|_\infty \to 0 \; as \; n \to \infty.$$

Step 2 The image of a set bounded by the operator F is a bounded set in PC(J;R).

Indeed, it suffices to show that for everything μ ∗ there exists a positive constant l such that for each y ∈ Bμ*= {y ∈ PC(J;R) : $\|y\|_\infty \leq$ μ ∗}we have $\|F(y)\|_\infty \leq l\}$.We have, for everything t ∈ J,

$$|F(y)(t)| \leq \left[\frac{l(m+1)(logT)^r}{\Gamma(r+1)} + \mathrm{ml}\,*\right] = \mathrm{l}$$

Step 3 F maps bounded sets into equicontinuous sets of PC(J;R).

Let τ_1, τ_2 ∈ J; $\tau_1 < \tau_2$, Bμ* is a bounded set which defined in 2rd step and let y ∈ Bμ* . So

$$|F(y)(\tau_2) - F(y)(\tau_1)| \leq \frac{M}{\Gamma(r+1)}[2[\log(\tau_2 - \tau_1)]^r + (\log\tau_2)^r - (\log\tau_1)^r] + \sum_{0<t_k<\tau_2-\tau_1} I_k(y(t_k^-)).$$

step 4 Apriori bounds

Let the set ε= { y ∈ PC(J;R) : y = τF(y) for all 0 < τ< 1} be bounded.

Let y ∈ ε, we have

$$y(t)=\left(\frac{\log(t-\mathrm{tk})}{log\mathrm{tk}}\right)^{\alpha-1}\left[\frac{\tau}{\Gamma(r)}\sum_{k=1}^{m}\int_{t_{k-1}}^{t_k}\left(\log\frac{t}{s}\right)^{r-1}\frac{f(s,y(s))}{s}ds + \sum_{k=1}^{m} I_k(y(t_k))\right]$$
$$+\frac{\tau}{\Gamma(r)}\int_{t_k}^{t}(\log\frac{t}{s})^{r-1}\frac{f(s,y(s))}{s}ds$$

for all t ∈J.

We use Schaefer's theorem assumptions, then we get

$$\|y\|_\infty \leq \frac{Mm(logT)^r}{\Gamma(r+1)} + \frac{M(logT)^r}{\Gamma(r+1)} + \text{ mM} *= \text{R}.$$

We prove that the set ε is bounded.

From Schaefer's fixed point theorem, we deduce that F has fixed points which are solutions of the problem (1.1)-(1.3).

In the following theorem we give an existence result for the problem (1.1)-(1.3) by applying the nonlinear alternative of Leray-Schauder type and for which the conditions (H4) and (H5) are weakened.

Theorem 3.5 Assume that (H2) and the following conditions hold:

(H6) There exist $\varphi_f \in C(J, R^+)$and ψ: [0;+∞) →(0;+∞) continuous and non-decreasing such that

$|f(t,u)| \leq \varphi_f(t)\psi(|u|)$for all t ∈ J; u ∈ R:

(H7) There exists $\psi *$: [0;+∞) →(0;+∞) continuous and nondecreasing such that

|Ik(u)| ≤ $\psi *$ $(|u|)$for all u ∈ R; i = 1,…, m.

(H8) There exists a number $\bar{M}$ > 0 such that

$$\frac{\bar{M}}{\psi(\bar{M})\frac{m\varphi_f^0(logT)^r}{\Gamma(r+1)}+\psi(\bar{M})\frac{\varphi_f^0(logT)^r}{\Gamma(r+1)}+m\,\psi * \bar{M}} > 1,$$

where

$$\varphi_f^0 = \sup\{\,\varphi_f(t): t \in J\}.$$

Then(1.1)-(1.3) has at least one solution on J.

Proof:

When we have shown that the operator F : $\bar{U}$ →PC(J;R) is continuous and completely continuous by the previous theorem.

1. For τ ∈[0; 1], let y such that, for each t ∈ J; we have y(t) = τ (Fy)(t). Then the Leray-Schauder theorem hypothesis we have for each t ∈ J,

$$\|y\|_\infty \leq \psi(\|y\|_\infty)\frac{m\varphi_f^0(logT)^r}{\Gamma(r+1)} + \psi(\|y\|_\infty)\frac{\varphi_f^0(logT)^r}{\Gamma(r+1)} + \text{ m}\psi * (\|y\|_\infty).$$

2. According to the Leray-Schauder theorem, there exists $\bar{M}$ > 0 such that $\|y\|_\infty \neq \bar{M}$. Let

U = {y ∈ PC(J;R) : $\|y\|_\infty < \bar{M}$}.

The choice of U, results in that there is no y ∈ ∂U such that y = τF(y) with $\tau \in$ (0; 1).

Therefore, after the Leray-Schauder nonlinear alternative, we deduce that F has a _xed point y ∈U which represents a solution of the problem (1.1)-(1.3).

4 AN EXAMPLE

In this section we give an example to illustrate the usefulness of our main results. Les us consider the impulsive fractional initial-value problem,

HDr y(t) =$\frac{e^{-t}|y(t)|}{(9+e^t)(1+|y(t)|)}$, for a.e. t ∈ $J = [1, e], t \neq \frac{3}{2}, 0 < r < 1,$ (4.1)

$\Delta y \backslash t = \frac{3}{2} = \frac{|y(\frac{3}{2})^-|}{3+|y(\frac{3}{2})^-|}$; k = 1,… , m; ; (4.2)

y(1) = 0: (4.3)

Set f(t; x) = $\frac{e^{-t}x}{(9+e^t)(1+x)}$; (t; x) ∈ J ×[0;+∞); and Ik(x) = $\frac{x}{3+x}$; x ∈ [0; +∞). Let x; y ∈ [0; +∞) and t ∈ J. Then we have

|f(t, x)- f(t, y)|≤ $\frac{1}{10}$ |x-y|.

Hence the condition (H1) holds with l = $\frac{1}{10}$.

Let x; y ∈ [0; +∞). Then we have

| Ik(x) − Ik(y) | ≤ $\frac{1}{3}$ |x-y|.

Hence the condition (H2) holds with l* = $\frac{1}{3}$.

3 . We shall check that condition (3.5) is satisfied

with T = e and m = 1. Indeed,

[$\frac{l(m+1)(log T)^r}{\Gamma(r+1)}$ + ml*]< $1\ iff\ \Gamma(r+1) > \frac{3}{10}$.; (4.4)

which is satisfied for some r ∈ (0; 1]. Then by Theorem (3.3), the problem (1.1)-(1.3) has a unique solution on [1; e] for values of r satisfying (4.4).

REFERENCES

[1] M. Benchohra, J. Henderson and S. K. Ntouyas, Impulsive Differential Equations and Inclusions, Hindawi Publishing Corporation, Vol. 2, New York, 2006.

[2] M. Benchohra and B. A. Slimani, Impulsive fractional differential equations, Electron. J. Diff. Equ. 2009 (2009), No. 10, 11 pp.

[3] J R. Graef, N. Guerraiche and S. Hamani, Initial value problem of fractional functional differential inclusions with Hadamard type derivative, Surveys in mathematics and its applications, Vol 13, p27-40, 2018.

[4] A. Granas and J. Dugundji, Fixed Point Theory. Springer-Verlag, New York, 2003.

[5] J. Hadamard, Essai sur l'_etude des fonctions données par leur developpement de Taylor, J. Math. Pure Appl. Ser. 8 (1892), 101-186.

[6] J. K. Hale and S. V. Lunel, Introduction to functional differential equations, Applied Mathematical Sciences, 99, Springer-Verlag, New York, 1993.

[7] S. Hamani,A. Hammou and J.Henderson, Impulsive Fractional Differential Equations Involving The Hadamard Fractional Derivative, Communication on applied nonlinear analysis, 24(2017), N 3,P 48-58.

[8] J. Henderson and C. Tisdell, Topological transversality and boundary value problems on time scales, J. Math. Anal. Appl. 289 (2004), 110-125.

[9] A. A. Kilbas, H. M. Srivastava and J. J. Trujillo, Theory and Applications of Fractional Differential Equations. North-Holland Mathematics Studies, 204. Elsevier Science B.V., Amsterdam, 2006.

COEXISTENCE OF TWO LIMIT CYCLES FOR A CLASS OF PLANAR DIFFERENTIAL SYSTEMS

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ABSTRACT

The existence of limit cycles is interesting and very important in applications. It is a key to understanding the dynamics of polynomial differential systems. It is difficult to determine the explicit expression of a limit cycle.

In this work, we investigate a class of a planar system and show that this system has two algebraic limit cycles around the same singular point. Furthermore, these limit cycles are explicitly given

Keywords: First integral, Periodic orbits, Algebraic limit cycle, Coexistence

INTRODUCTION AND PRELIMINARIES

A polynomial differential systems on the plane are systems of the form

$$\begin{cases} \dot{x} = \dfrac{dx}{dt} = P(x, y), \\ \dot{y} = \dfrac{dy}{dt} = Q(x, y), \end{cases} \qquad (1)$$

where P and Q are two coprime polynomials of $\mathbb{R}[x, y]$ and the derivatives are performed with respect to the time variable. By definition, the degree of the system (1) is the maximum of the degrees of the polynomials P and Q.

The algebraic curve $U(x, y) = 0$ is called an invariant curve for system (1) if there exists a polynomial $K(x, y)$ (*called the cofactor*) such that

$$P(x, y)\frac{\partial U}{\partial x}(x, y) + Q(x, y)\frac{\partial U}{\partial y}(x, y) = K(x, y)U(x, y).$$

We recall that in the phase plane, a *limit cycle* of system (1) is an isolated periodic solution in the set of all its periodic solutions. If limit cycle contained in the zero set of invariant algebraic curve of the plane, then we say that it is *algebraic*, otherwise it is called *non-algebraic*.

In the qualitative theory of planar polynomial differential systems [5], one of the most important topics is related to the second part of the unsolved Hilbert 16th problem, concerned essentially by the number $H(n)$ of limit cycles of (1) and their positions in the phase space. There is an extensive literature on that subject, most of it deals essentially with detection, number and stability of limit cycles.

Another interesting problem is to give an explicit expression of a limit cycle. Until recently, the only limit cycles known in an explicit way were algebraic (see, for example, [1, 2, 3, 4, 7] and references therein).

To my knowledge, if a system admits more than one algebraic limit cycle, each of these cycles surrounds a singular point different from the other points. For example, Bendjeddou and Cheurfa [3] studied a class of quartic differential system and showed under certain conditions, the existence of up to four limit cycles but each cycle surrounds a singular point different from the others.

In this work, we are interested on the differential system of the form

$$\begin{aligned} \dot{x} &= P_5(x,y) = abx + \left(x^2+y^2\right)\left(-(a+b)x - 2y + x\left(x^2+y^2\right)\right), \\ \dot{y} &= Q_5(x,y) = aby + \left(x^2+y^2\right)\left(2x - (a+b)y + y\left(x^2+y^2\right)\right). \end{aligned} \qquad (2)$$

where a and b are reals constants such that $a > b > 0$. Within this class, we prove the existence of two algebraic limit cycles surrounding the same singular point, moreover these limit cycles are explicitly given.

Proposition1: *The origin is the only singular point of the system (2) and is a unstable node.*

Proof : We have

$$yP_5(x,y) - xQ_5(x,y) = -2\left(x^2+y^2\right)^2,$$

thus the origin is the unique singular point of this system. Moreover the unique eigenvalue of the associated linearized system is $\lambda = ab > 0,$ then the origin is unstable node.

THE MAIN RESULT

Our main result is as follows

Theorem1: *The system (2) admits the two circles* $(\Gamma_1)\ :\ U(x,y) = x^2+y^2-a=0$ *and* $(\Gamma_2)\ :\ V(x,y) = x^2+y^2-b=0$ *as limit cycles. Moreover* (Γ_1) *is an unstable,* (Γ_2) *is a stable and* (Γ_2) *lies inside* (Γ_1).

Proof: It is easy to check that the system (2) possesses (Γ_1) and (Γ_2) as invariant algebraic curves. The associated cofactors are respectively given

$$K_1 = 2\left(x^2+y^2\right)\left(x^2+y^2-b\right)$$

and

$$K_2 = 2(x^2 + y^2)(x^2 + y^2 - a).$$

As the two circles (Γ_1) and (Γ_2) do not pass through the origin, thus they are periodic solutions of the system (2).

Let T_1 and T_2 denotes be the periods of (Γ_1) and (Γ_2) respectively. To show that (Γ_1) and (Γ_2) are limit cycles, it is sufficient to check that

$$I(\Gamma_1) = \int_0^{T_1} \operatorname{div}(x(t), y(t))dt \neq 0$$

and

$$J(\Gamma_2) = \int_0^{T_2} \operatorname{div}(x(t), y(t))dt \neq 0.$$

According to theorem 3 of [7], we have

$$I(\Gamma_1) = \int_0^{T_1} \operatorname{div}(x(t), y(t))dt = \int_0^{T_1} K_1(x(t), y(t))dt$$

and

$$J(\Gamma 2) = \int_0^{T_2} \operatorname{div}(x(t), y(t))dt = \int_0^{T_2} K_2(x(t), y(t))dt.$$

Therefore

$$\begin{aligned} I(\Gamma_1) &= \int_0^{T_1} 2(x^2 + y^2)(x^2 + y^2 - b)dt \\ &= \int_0^{T_1} 2(x^2 + y^2)(a - b)dt > 0 \end{aligned}$$

and

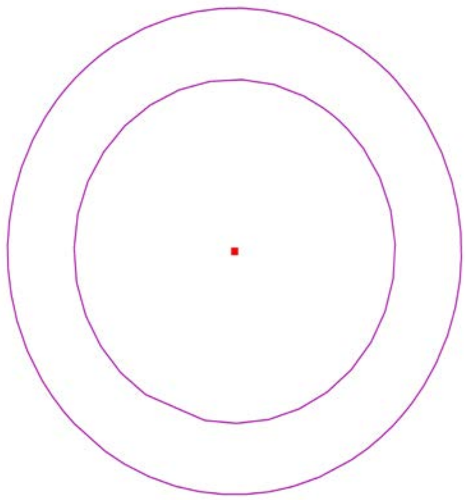
$$\begin{aligned} J(\Gamma 2) &= \int_0^{T_2} 2(x^2 + y^2)(x^2 + y^2 - a)dt \\ &= \int_0^{T2} 2(x^2 + y^2)(b - a)dt < 0. \end{aligned}$$

Consequently, (Γ_1) and (Γ_2) define respectively an unstable limit cycle and a stable limit cycle for system (2). This complete the proof of theorem1.

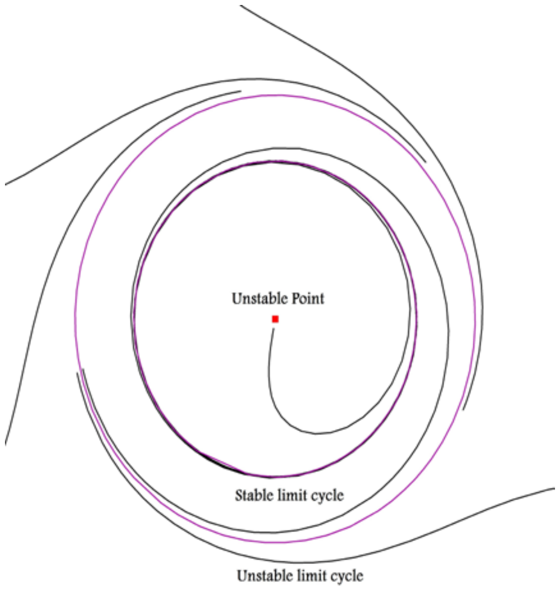
Example1: *In the system (2), we take* $a = 1$ *and* $b = 2$, *we obtain*

$$\begin{aligned} \dot{x} &= 2x + (x^2 + y^2)(-3x - 2y + x(x^2 + y^2)), \\ \dot{y} &= 2y + (x^2 + y^2)(2x - 3y + y(x^2 + y^2)). \end{aligned} \qquad (3)$$

which has two limit cycles (Γ_1) : $x^2 + y^2 - 1 = 0$ *and* (Γ_2) : $x^2 + y^2 - 2 = 0$ *Moreover, the circle* (Γ_2) *lies inside* (Γ_1) *as shown on the Poincaré disc in* ***Figure 1***.



(I)



(II)

Figure 1 : (I) Represents the two limit cycles of the system (3) and (II) its phase portrait on the Poincaré disc

CONCLUSION

In this work, we have studied a quintic system and we have shown that it admits two circles as limit cycles surrounding the unique singular point, one inside the other (coexistence)

Finally, it is of interest to extend this study by answering to the following questions: Is there a cubic or quartic system that exhibit two algebraic limit cycles surrounding the same singular point?

REFERENCES

1. Bendjeddou, A. Berbache and R. Cheurfa, A class of Kolmogorov system with exact algebraic limit cycle, Int. J. of Diff. Equa. Appli, Vol. 14, No.3 (2015), 159-165.

2. Bendjeddou, R. Cheurfa, On the exact limit cycle for some class of planar differential systems, Nonlinear diff. equ. appl. 14 (2007), 491--498.

3. A. Bendjeddou and R. Cheurfa, Cubic and quartic planar differential system with exact algebraic limit cycles, Elect. J. of Diff. Equ., Vol 2011 (2011), No15, 1-12.

4. S. Benyoucef, A. Bendjeddou, Kolmogorov system with explicit hyperbolic limit cycle, J. Sib. Fed. Univ. Math. Phys., Vol. 10, No.2 (2017), 216--222.

5. F. Dumortier, J. Llibre and J. Artés, Qualitative Theory of Planar Differential Systems, (Universitex) Berlin, Springer (2006).

6. H Giacomini, J Llibre, M Viano, On the nonexistence, existence and uniqueness of limit cycles, Nonlinearity 9 (1996) 501—516

7. J. Giné, M. Grau, A note on : Relaxation oscillator with exact limit cycles, J. of Math. Anal. and Appl., 224 (2006), no 1, 739--745.

8. J. Llibre, J. Yu, X. Zhang, On the limit Cycle of the Polynomial Differential Systems with a Linear Node and Homogeneous Nonlinearities, Inter. of Bifu. and Chaos, Vol. 24, No. 5 (2014) 1450065.

9. L. Perko, Differential equations and dynamical systems, Third edition.Texts in Applied Mathematics, 7. Springer-Verlag, New York, 2001.

SURFACES WITH DENSITY IN MINKOWSKI 3-SPACE

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ABSTRACT

In this work, firstly we give some basic notations, definitions, theorems and results about surface with density in Euclidean and Minkowski 3-space. After that, we give a summary of informations about revolution surface and translation surface in Minkowski 3-space. Later, we write the equation of minimal surfaces in Minkowski 3- space with linear density (in the case $\varphi(x,y,z) = x$, $\varphi(x,y,z) = y$ and $\varphi(x,y,z) = z$), and we characterize some solutions of the equation of minimal graphs in Minkowski 3-space with linear density $\Psi= e^{\varphi}$. Moreover, we write the φ-Gauss curvature and the φ - mean curvature formulae of the some revolution surfaces in Minkowkski 3-space with radial density $e^{-a\rho^2+c}$. After this, we give some example and draw the graphs of φ- minimal surfaces for some special cases via Matlab program.

Key Words: Surfeces With Density, Translation Surfaces, Minimal Surface, Minkowski 3-Space, Graphical Surface

TUBULAR SURFACES CONSTRUCTED BY SPHERICAL INDICATRICES WITH DENSITY

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ABSTRACT

In this work, firstly we give some basic notations, definitions, theorems and results about surface with density in Euclidean and Minkowski 3-space. After that, we give a summary of informations about canal and tubular surfaces in Euclidean 3 space. Later, we give the parametrisations of tubular surface constructed by spherical indicatrices of any spatial curve in Eucidean 3- space. In this work we construct the tubular surface according to the alternative moving frame {N,C,W}. Moreover, we write the φ-Gauss curvature and the φ - mean curvature formulae of the tubular surfaces which constructed by spherical indicarices in Euclidean 3-space with some density. After this, we give φ-flat and φ-minimal equations of these surfaces. Moreover, we give the conditions for being φ-flat and φ-minimal of these surface

Key Words: Surfaces With Density, Tubular Surfaces, Minimal Surface, Alternative Moving Frame.

ORDERS OF SOLUTIONS OF FRACTIONAL DIFFERENTIAL EQUATION

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ABSTRACT

We study the solutions of the fractional differential equation

$$D^{(\alpha)}f + A(z)D^{(\beta)}f + B(z)f = 0$$

where $D^{(\alpha)}$, and $D^{(\beta)}$, are the Caputo fractional derivatives of orders and z is complex number, $A(z)$, $B(z)$ be entire functions. We find conditions on the coefficients so that every solution that is not identically zero has infinite order.

Key words. The Caputo fractional derivative, entire function, infinite order, complex domain.

INTRODUCTION

Recently, the complex modelings of phenomena in nature and society have been the object of several investigations based on the methods originally developed in a physical context. These systems are the consequence of the ability of individuals to develop strategies. They occur in kinetic theory [1], complex dynamical systems [19], chaotic complex systems and hyperchaotic complex systems [25], and the complex Lorenz-like system which has been found in laser physics while analyzing baroclinic instability of the geophysical flows in the atmosphere (or in the ocean) [22, 26]. Sainty [23] considered the complex heat equation using a complex valued Brownian. A model of complex fractional equations is introduced by Jumarie [14, 15, 16, 17], using different types of fractional derivatives. Baleanu et al. [2, 3, 18], imposed several applications of fractional calculus including complex modelings. The author studied various types of fractional differential equations in complex domain such as the Cauchy equation, the diffusion equation and telegraph equations [9, 10, 11, 12, 13]. Transformis a significant technique to solve mathematical problems.Many useful transforms for solving various problems appeared in open literature such as wave transformation, the Laplace transform, the Fourier transform, the Bücklund transformation, the integral transform, the local fractional integral transforms and the fractional complex transform (see [5, 21])

In this section [24], we introduce some notations and definitions for fractional

operators (derivative and integral) in the complex z-plane C as follows.

Definition 1. The fractional derivative of order α is defined, for a function f(z), by

$$D^{(\alpha)}f(z) = \frac{1}{\Gamma(1-\alpha)}\int_0^z \frac{f(\xi)d\xi}{(z-\xi)^{\alpha}}, 0 \leq \alpha < 1,$$

where the function f is analytic in a simply-connected region of the complex plane containing the origin, and the multiplicity of $(z-\xi)^{\alpha}$ is removed by requiring $\log(z-\xi)^{\alpha}$ to be real when $(z-\xi)>0$.

Throughout this paper, we assume that the reader is familiar with the fundamental

results and the standard notations of the Nevanlinna value distribution theory of meromorphic functions (see [8]). Let $\rho(f)$ denote the order of an entire function f, that is

$$\rho(f)=\overline{\lim_{r\rightarrow+\infty}}\frac{\log T(r,f)}{\log r},$$

where $T(r; f)$ is the Nevanlinna characteristic function of f (see [8]).

REFERENCES

[1] C. Bianca, On set of nonlinearity in thermostatted active particles models for complex systems. Nonlinear Anal., Real World Appl. 13, 2593-2608 (2012)

[2] D. Baleanu, K. Diethelm, E. Scalas, J.J Trujillo, Fractional Calculus Models and Numerical Methods (Series on Complexity, Nonlinearity and Chaos). World Scientific, Singapore (2012)

[3] D. Baleanu, J.J Trujillo, On exact solutions of a class of fractional Euler-Lagrange equations. Nonlinear Dyn. 52, 331-335 (2008)

[4] A.C. Fowler, J.D. Gibbon, M.J. McGuinness, The complex Lorenz equations. Physica D 4, 139-163 (1982)

[5] P.R Gordoa, A. Pickering, Z.N. Zhu, Bücklund transformations for a matrix second Painlev equation. Phys. Lett. A 374(34), 3422-3424 (2010)

[6] G.G. Gundersen, Estimates for the logarithmic derivative of a meromorphic function, plus similar estimates, J. London Math. Soc. (to appear).

[7] G. Gundersen, Finite order solutions of second order linear differential equations, Trans. Amer. Math. Soc. 305 (1988), pp. 415-429.

[8] W. K. Hayman, Meromorphic functions, Clarendon Press, Oxford, 1964.

[9] R.W. Ibrahim, M. Darus, Subordination and superordination for univalent solutions for fractional differential equations. J. Math. Anal. Appl. 345, 871-879 (2008)

[10] R.W. Ibrahim, On solutions for fractional diffusion problems. Electron. J. Differ. Equ. 147, 1-11 (2010)

[11] R.W. Ibrahim, Existence J. Math. Anal. Appl. 380, 232-240 (2011) and uniqueness of holomorphic solutions for fractional Cauchy problem.

[12] R.W. Ibrahim, Ulam stability for fractional differential equation in complex domain. Abstr. Appl. Anal. 2012, Article ID 649517 (2012).

[13] R.W. Ibrahim, On holomorphic solution for space and time fractional telegraph equations in complex domain. J. Funct. Spaces Appl. 2012, Article ID 703681 (2012).

[14] G. Jumarie, Fractional Brownian motion with complex variance via random walk in the complex plane and applications. Chaos Solitons Fractals 11, 1097-1111 (2000)

[15] G. Jumarie, Schrodinger equation for quantum fractal space-time of order n via the complexvalued fractional Brownian motion. Int. J. Mod. Phys. A 16, 5061-5084 (2001)

[16] G. Jumarie, Fractional Brownian motions via random walk in the complex plane and via fractional derivative. Comparison and further results. Chaos Solitons Fractals 22, 907-925(2004)

[17] G. Jumarie, Fractionalization of the complex-valued Brownian motion of order n using Riemann-Liouville derivative. Applications to mathematical finance and stochastic mechanics. Chaos Solitons Fractals 28, 1285-1305 (2006)

[18] R. Magin, X. Feng, D. Baleanu, Solving the fractional order Bloch equation. Concepts Magn. Reson., Part A 34A, 16-23 (2009)

[19] G. Mahmoud, E. Mahmoud, Modified projective Lag synchronization of two nonidentical hyperchaotic complex nonlinear systems. Int. J. Bifurc. Chaos 2(8), 2369-2379 (2011)

[20] G. Mahmoud, E. Mahmoud, M. Ahmed, On the hyperchaotic complex Lü system. Nonlinear Dyn. 58, 725-738 (2009)

[21] R. Molliq, B. Batiha, Approximate analytic solutions of fractional Zakharov-Kuznetsov equations by fractional complex transform. Int. J. Eng. Technol. 1(1), 1-13 (2012)

[22] A. Rauth, L. Hannibal, N.B. Abraham, Global stability properties of the complex Lorenz model. Physica D 99, 45-58 (1996)

[23] P. Sainty, Contraction of a complex-valued fractional Brownian of order n. J. Math. Phys. 33(9), 3128-3149 (1992)

[24] H.M. Srivastava, S.Owa, Univalent Functions, Fractional Calculus, and Their Applications. Wiley, New York (1989)

[25] M. Tsuji, Differential theory in modern function theory, Chelsea, New York, 1975, reprint of the 1959 edition.

[26] L. Yang, Value distribution theory, Springer-Verlag Science Press, Berlin-. Beijing. 1993

EXISTENCE RESULT OF A FRACTIONAL DIFFERENTIAL EQUATION OF HADAMARD TYPE WITH INTEGRAL BOUNDARY VALUE CONDITIONS

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ABSTRACT

In this woek, we establish the existence of solutions for a fractional differential equations with Hadamard fractional integral boundary condition. Our main result are obtained by using generalization of Darbo's fixed point theorem combined with the technique of measures of noncompactness in the Banach algebras. An example is provided to illustrate our main results.

Keywords: Integral boundary value conditions; Measure of noncompactness; Hadamard fractional derivative; upper semicontinuous function.

REFERENCES

1 A. Kilbas, H. M. Srivastava, J. J Trujillo, Theory and Applications of Fractional Differential Equations, vol. 204 of North-Holland Mathematics Sudies Elsevier Science B.V. Amsterdam the Netherlands, 2006.

2 Ahmad, S.K. Ntouyas, J. Tariboon, Existence results for mixed Hadamard and Riemann-Liouville fractional integro-differential equations, Adv. Difference Equ, 2015 (2015), 293.

3 U. C¸ akana, I. Ozdemir, Existence of nondecreasing solutions of some nonlinear integral equations of fractional order, J. Nonlinear Sci. Appl. 8 (2015), 1112-1126.

4 U. C¸ akana, I. Ozdemir, Applications of Measure of Noncompactness and Darbo's Fixed Point Theorem to Nonlinear Integral Equations in Banach Spaces, Numer. Funct. Anal. Optim. 38 (2017), 641-673.

5 S. Dudek, Measures of noncompactness in a Banach algebra and their applications, J. Math. Anal. 40 (2017), 69-84.

LOGARITHMIC DERIVATIVE NEAR A SINGULAR POINT AND APPLICATIONS IN LINEAR DIFFERENTIAL EQUATIONS

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ABSTRACT

The logarithmic derivative play an important role in the study of the growth and oscillation of solutions of differential equations in the complex plane and in the unit disc. In this talk, we will provide new estimates of logarithmic derivatives around an isolated finite singular point by making use a suitable conformal mapping and an addaptation notions of Nevanlinna theory of meromorphic function and Wiman-Valiron theory of entire function. As applications, we investigate the growth of solutions of certain types of linear differential equations with non meromorphic coefficients.

Keywords: Nevanlinna theory, Wiman-Valiron theory, logarithmic derivative, linear differential equations, growth of solutions.

REFERENCES

[1] Fettouch H. and Hamouda S., Growth of local solutions to linear differential equations around an isolated essential singularity, Electron. J. Differential Equations, Vol 2016 (2016), No. 226, pp. 1-10.

[2] Gundersen G. G., Estimates for the logarithmic derivative of a meromorphic function, plus similar estimates, J. Lond. Math. Soc. (2), 37 (1988), 88-104.

[3] I. Laine, Nevanlinna theory and complex differential equations, W. de Gruyter, Berlin, 1993.

DERECELENDİRİLMİŞ ASAL İDEALLERİN GENELLEŞTİRİLMESİ ÜZERİNE BİR NOT

A NOTE ON A GENERALIZATION OF GRADED PRIME IDEALS

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ÖZET

Bu çalışmada derecelendirilmiş asal ideallerin bir genellemesi tanıtılacaktır. Bu genelleme yardımıyla derecelendirilmiş asal idealler, derecelendirilmiş zayıf asal idealler gibi çeşitli ideallerin arasındaki ilişkiler ortaya konulacaktır.

Anahtar Kelimeler: derecelendirilmiş asal ideal, derecelendirilmiş zayıf asal ideal, derecelendirilmiş 2-yutan ideal, derecelendirilmiş yarı asalımsı ideal.

ABSTRACT

In this paper, a generalization of graded prime ideals will be introduced. With the help of this generalization, the relationships between various ideals such as graded prime ideals and graded weakly prime ideals will be revealed.

Keywords: graded prime ideal, graded weakly prime ideal, graded 2-absorbing ideal, graded quasi primary ideal.

REFERENCES

[1] Uregen, R. N., Tekir, U., Shum, K. P., & Koc, S. (2019). On Graded 2-Absorbing Quasi Primary Ideals. Southeast Asian Bulletin of Mathematics, 43(4), 601-613.

[2] D. D. Anderson and M. Bataineh, Generalizations of Prime Ideals, Communications in Algebra, 36 (2008), 686–696.

[3] S. E. Atani, On Graded Weakly Prime Ideals, Turkish Journal of Mathematics, 30 (2006), 351–358.

[4] M. Refai and K. Al-Zoubi, On Graded Primary Ideals, Turkish Journal of Mathematics, 28, (2004), 217–229.

STABILITY OF SOLUTIONS OF INITIAL VALUE PROBLEM FOR A CLASS OF STOCHASTIC FRACTIONAL DIFFERENTIAL EQUATION WITH NOISE

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ABSTRACT

In this work, we will introduce a fractional Duhamel principle and use it to establish the well boudedness and stability of a mild solution to fractional stochastic equation with initial data.

Keywords: Stochastic equation, Fractional derivative, Mild solution, Stability.

REFERENCES

A. Bensoussan, R. Temam, Equations stochastiques du type Navier-Stokes, J. Functional Analysis, 13 (1973), 195-222.

F. Biagini, Y. Hu, B. Oksendal, T. Zhang, Stochastic calculus for fractional Brownian motion and applications, Springer (2008).

Y.Y. Gambo, F. Jarad, D. Baleanu, T. Abdeljawad, On Caputo modification of the Hadamard fractional derivatives, Adv. Diff. Equ. 2014 (1) (2014) 112.

T. E. Govindan, Stability of mild solutions of stochastic evolutions with variable decay, Stoch. Anal. Appl. 21(2003) 1059-1077.

L. Hörmander, The Analysis of Linear Partial Differential Operators. Springer, Berlin (2005).

A. A. Kilbas, H. M. Srivastava, J. J. Trujillo, Theory and Applications of Fractional Differential Equations. Elsevier, Amsterdam (2006) .

K. Liu, Stability of Infnite Dimensional Stochastic Differential Equations with Applications,

Chapman Hall, CRC, London, 2006.

J. Luo, Fixed points and exponential stability of mild solutions of stochastic partial differential equations with delays, J. Math. Anal. Appl. 342 (2) (2008), 753-760.

B. Oksendal, Stochastic Differential Equations 6th ed. Springer: New York, 2005.

A. Seemab, M. Rehman, A note on fractional Duhamel\textquoteright s principle and its application to a class of fractional partial differential equations, Applied Mathematics Letters 64 (2017) 8\textendash 14.

S. Umarov, On fractional Duhamel\textquoteright s principle and its applications, J. Differential Equations 252 (2012), 5217\textendash 5234.

Y. Wen, X. F. Zhou and J. Wang, Stability and boundedness of solutions of the initial value problem for a class of time-fractional diffusion equations, Advances in Difference Equations, (2017), 10 pages.

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